Noncooperative Bargaining with Nonanonymous Outside Options*

Gökhan Buturak†
Stockholm School of Economics

November 24, 2008

Abstract

In this note, we extend the standard noncooperative bargaining game with two-sided outside options to the case where players’ payoffs resulting from opting out (what we call as outside options) depend on who among the players opts out first. With such an extension, we show that when each player prefers the outside option of the other, the game may admit a continuum of equilibria (some of which are with delay) though the players are locked (or compelled to stay) in the game, not by assumption, but as a consequence of the payoff structure.

Moreover, we document other equilibria under different payoff structures and show in general that the extension of the standard alternating offers bargaining game with nonanonymous outside options enriches the set of equilibria.

1 Introduction

This note extends the standard bargaining game with alternating offers by introducing nonanonymous outside options. By nonanonymous outside options, we mean in case either of the players leaves the table, players receive disagreement payoffs which are not necessarily equal to the payoffs they get when the other leaves the table. In other words, who leaves the table first matters in terms of payoff consequences.

The extension of the strategic bargaining framework with nonanonymous outside options makes sense in real life since in most of the bargaining and negotiation cases the consequences of opting out decisions of the bargaining parties are not symmetric. Kıbrıs and Tapkı [4] give the example of bargaining between firms and labour unions.

---

*I am grateful to Mark Voorneveld and Tore Ellingsen for their helpful comments. The usual disclaimer applies. Financial support from the Wallander/Hedelius Foundation is gratefully acknowledged.
†Department of Economics, Stockholm School of Economics, P.O. Box 6501 SE-11383 Stockholm, Sweden. E-mail: gokhan.buturak@hhs.se. Tel: +46-8-736 9642.
where the event of disagreement is called 'lockout' in case the firm opts out whereas it is called 'strike' if the labour union opts out and these two outcomes typically have different consequences\textsuperscript{1}. In the United States, according to U.S. Labour Law, an employer may only hire temporary replacements during a lockout. On the other hand, due to Supreme Court’s 1938 Mackay opinion, in a strike, unless it is an unfair labour practice strike, an employer may legally hire permanent replacements, but cannot fire the strikers\textsuperscript{2} (see e.g. Getman and Kohler [3], Yates [9]). Moreover, in some U.S. states, employees who are locked out are eligible to receive unemployment benefits, but are not eligible for such benefits during a strike (Yates [9]). Apparently, the legislator discriminates the status of the bargaining parties according to who opts out first in order to compel the bargaining parties to remain at the bargaining table. As a result, many American employers have historically been reluctant to impose lockouts, instead seeking for the employees to strike.

Another example might be the peace agreement negotiations between two countries. In the eyes of third parties, whichever leaves the table is usually considered to be the ‘bad guy’ since it simply indicates reluctance to put continued effort to maintain peace and show any intention of agreement. Even in a simple disputation case, adversaries have the option of using violence, in which case the one who initiated violence gets convicted and is considered to be guilty since he initiated the violence, whereas the other remains to be in a better situation. In marriage, divorce can be considered to be the disagreement alternative of both the husband and wife. In some societies, whoever initiates the divorce process is perceived as cantankerous and responsible for the breakdown of the marriage.

In the above examples, each party prefers the outside option where the other party opts out to the one where he opts out. But there are of course situations in which case both parties prefer one of the outside options. An example to this may be the case of two conflicting countries where either of the country’s opting out decision refers to attacking the other. One may have heavy armament (e.g. nuclear ammunition) compared to the other, so that both countries prefer the country with heavy armament not to use nuclear power and to back off. There is another possibility where each party prefers his own outside option, but such a scenario is trivial, which we will skip.

As is well known, noncooperative bargaining theory was initiated by Nash [5] in the early 1950s, which introduces the ‘Nash Demand Game’, and gained impetus by Rubinstein’s [8] seminal paper. In Rubinstein’s [8] model, players are locked in the bargaining game and have no opportunity to opt out. The models following Rubinstein’s [8], which extend the standard alternating offers model by assuming that the bargainers are not locked in and can opt out of the bargaining process without agreement, use the term out-

\textsuperscript{1}Though not in all countries.
\textsuperscript{2}In practice, permanent replacement is not much different than giving a notice.
side option to indicate the best outside alternative that a player can get if he withdraws unilaterally from the bargaining process.

Binmore, Rubinstein, and Wolinsky [1] first mention the possibility of outside options to be nonanonymous, but they don’t elaborate it:

"An outside option is defined to be the best alternative that a player can command if he withdraws unilaterally from the bargaining process. Usually it is assumed that there exists an outcome $e \in X$ (the "outside option point") that results if either bargaining party withdraws from the bargaining process, although a more general description might specify two outcomes, depending on who withdraws."

Later, Corominas-Bosch [2] extends the model of Binmore, Rubinstein, and Wolinsky [1] with asymmetric exogenous breakdown outcomes. In her model, there is an exogenous probability of the bargaining process to break down, and in case the bargaining parties cannot reach an agreement before the breakdown occurs, the parties receive nonzero payoffs according to who rejected lastly before the exogenous breakdown occurs. This model resembles the model in this note, but differs in two respects. First, it handles the nonanonymity of the exogenous breakdown outcomes, not the outcomes that occur by the opting out decision of either of the bargaining parties; and second, players don’t discount the future payoffs. Kbris and Tapki [4] extend the axiomatic bargaining framework with asymmetric, or "nonanonymous" in their words -hence we use in this note-, disagreement outcomes. Their model uses an extension of the Nash bargaining solution, and as in all the other axiomatic bargaining literature, there is no clear explanation of how the bargaining parties reach agreement.

The closest paper to this note is Ponsati and Sákovics [7], in which the Stáhl-Rubinstein bargaining framework is extended by two-sided outside options. But they consider the outside options to be a status-quo position for the bargaining parties independent from the whole bargaining process and the identity of who opts out first. Instead, as we have explained, we let the payoffs to opting out decisions depend on the identity of the disagreeer, which, we show, changes the set of equilibria.

The plan of the paper is as follows: In the following section, we explain the model. In the third section, we document some equilibria of the game described in the second section under two different cases: In subsection 3.1, we consider the case where one of the outside options is preferred by both players. In subsection 3.2, we deal with the case where the parties prefer distinct outside options. We conclude in section 4.

---

3 Note that outside option is one of the many possible forms of disagreement outcomes.
2 The Model

Player 1 and Player 2 bargain over the division of a fixed surplus, normalized to one. Time runs in discrete periods of equal length, indexed by $t = 0, 1, \ldots$. Players make alternating offers in an infinite horizon. In even (odd) periods Player 1 (Player 2) makes an offer. The other party may accept, thus terminating the game with agreement at the proposed shares. Alternatively, if the proposal is rejected, either of the two parties may leave the table and take their outside options, where their outside options depend on who leaves the table. If the offer is rejected but neither player leaves the table, then the game continues to the following round. We assume that infinite paths of play without reaching an agreement are the least-preferred outcomes. Player $i$ discounts future payoffs with $\delta_i \in (0, 1)$. If the game ends with agreement on shares $(\alpha, 1 - \alpha)$ in period $t$, then Player 1’s payoff is $\alpha \delta_1^t$, while Player 2’s payoff is $(1 - \alpha) \delta_2^t$. Similarly, if Player 1 alone decides to opt out in period $t$, then their payoffs are $(d_1^1 \delta_1^t, d_2^2 \delta_2^t)$, while if Player 2 alone decides to opt out in period $t$, then their payoffs are $(d_2^1 \delta_1^t, d_2^2 \delta_2^t)$. And finally, if both opt out simultaneously in period $t$, then their payoffs are $(d_1^1 \delta_1^t, d_1^2 \delta_2^t)$. Moreover, we use the letters $\psi$ and $\omega$ as auxiliary payoffs, prescribed by the (candidate) equilibria, to Player 1 when Player 1 and Player 2, respectively, offer.

3 Solution

As we have indicated in the Introduction section, different payoff structures of the outside options fall into different cases. Before we start to analyse the model case-by-case, regardless of which outside option each player prefers, we note that there exists a continuum of equilibria when the players’ own outside options are not so satisfactory or both players are patient enough, i.e. when $d_{11} \leq \frac{\delta_1 - \delta_1 \delta_2}{1 - \delta_1 \delta_2}$, and $d_{22} \leq \frac{\delta_2 - \delta_1 \delta_2}{1 - \delta_1 \delta_2}$. The intuition behind this result is that when both players see that both of them are patient enough and opting out is not worthy for both, even if each gets 1 when the other opts out, they choose to agree on a reasonable share and avoid going into a series of nonserious offers.

**Proposition 1** If $d_{11} \leq \frac{\delta_1 - \delta_1 \delta_2}{1 - \delta_1 \delta_2}$, and $d_{22} \leq \frac{\delta_2 - \delta_1 \delta_2}{1 - \delta_1 \delta_2}$, then the bargaining game has subgame perfect equilibria that result in immediate and efficient agreement. In particular, let $\psi, \omega \in$
The following strategies constitute a subgame perfect equilibrium where players agree upon \((\psi, 1 - \psi)\) in the initial period:

- **Player 1 always proposes** \((\psi, 1 - \psi)\), **rejects a proposal** \((\tilde{\psi}, 1 - \tilde{\psi})\) if and only if \(\tilde{\psi} < \psi\), **and never opts out**;

- **Player 2 always proposes** \((\omega, 1 - \omega)\), **rejects a proposal** \((\tilde{\psi}, 1 - \tilde{\psi})\) if and only if \(\tilde{\psi} > \psi\), **and never opts out**.

**Proof.**

**CASE 1:** Consider a subgame starting in an even period \(t\).

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in agreed-upon shares \((\psi, 1 - \psi)\). Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer \((\tilde{\psi}, 1 - \tilde{\psi})\) with \(\tilde{\psi} \leq \psi\) of Player 1 in some even period \(\tilde{t} \geq t\). As \(\tilde{\psi} \leq \psi\), this is not profitable;

2. After an accepted offer \((\omega, 1 - \omega)\) of Player 2 in some odd period \(\tilde{t} > t\). As \(\psi, \omega \in [\delta_1 - \delta_1 \delta_2, 1 - \delta_2 \delta_1]\), it follows that \(\delta_1 \omega \leq \psi\), so this is not profitable;

3. In disagreement outcome \(d_1\). As \(d_{11} \leq \psi\) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in agreed-upon shares \((\psi, 1 - \psi)\). Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer \((\psi, 1 - \psi)\) of Player 1 in some even period \(\tilde{t} > t\). Clearly, deferring this payoff is not profitable;

2. After an accepted offer \((\tilde{\omega}, 1 - \tilde{\omega})\) with \(\tilde{\omega} \geq \omega\) of Player 2 in some odd period \(\tilde{t} > t\). As \(\psi, \omega \in [\delta_1 - \delta_1 \delta_2, 1 - \delta_2 \delta_1]\), it follows that \(\delta_2 (1 - \tilde{\omega}) \leq \tilde{\omega} (1 - \omega) \leq 1 - \psi\), so this is not profitable;

3. In disagreement outcome \(d_2\). As \(d_{22} \leq (1 - \psi)\) by assumption, this is not profitable.

**CASE 2:** Consider a subgame starting in an odd period \(t\).

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in agreed-upon shares \((\omega, 1 - \omega)\). Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:
1. After an accepted offer \((\hat{\psi}, 1 - \hat{\psi})\) with \(\hat{\psi} \leq \psi\) of Player 1 in some even period \(\tilde{t} > t\). As \(\psi, \omega \in [\frac{\delta_1 - \delta_1^2}{1 - \delta_1 \delta_2}, \frac{1 - \delta_2}{1 - \delta_1 \delta_2}]\), it follows that \(\delta_1 \hat{\psi} \leq \omega\), so this is not profitable.

2. After an accepted offer \((\omega, 1 - \omega)\) of Player 2 in some odd period \(\tilde{t} > t\). Clearly, deferring this payoff is not profitable.

3. In disagreement outcome \(d_1\). As \(d_{11} \leq \omega\) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in agreed-upon shares \((\omega, 1 - \omega)\). Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer \((\psi, 1 - \psi)\) of Player 1 in some even period \(\tilde{t} > t\). As \(\psi, \omega \in [\frac{\delta_1 - \delta_1^2}{1 - \delta_1 \delta_2}, \frac{1 - \delta_2}{1 - \delta_1 \delta_2}]\), it follows that \(\delta_2 (1 - \psi) \leq (1 - \omega)\), so this is not profitable.

2. After an accepted offer \((\tilde{\omega}, 1 - \tilde{\omega})\) with \(\tilde{\omega} \geq \omega\) of Player 2 in some odd period \(\tilde{t} \geq t\). As \(1 - \tilde{\omega} \leq 1 - \omega\), this is not profitable.

3. In disagreement outcome \(d_2\). As \(d_{22} \leq (1 - \omega)\) by assumption, this is not profitable.

As in Ponsati and Sákovics [7], we have the immediate corollary:

**Corollary 1** The bargaining game described above admits a continuum of equilibria with immediate efficient agreement that gives Player 1 a share \(\psi \in [\frac{\delta_1 - \delta_1^2}{1 - \delta_1 \delta_2}, \frac{1 - \delta_2}{1 - \delta_1 \delta_2}]\) even if the players’ own disagreement payoffs are zero; i.e. \(d_{ii} = 0\) for \(i \in \{1, 2\}\); in particular when \(d_1\) and \(d_2\) approach to \((0, 0)\).

### 3.1 The case where each player prefers the outside option of the other to his/her outside option

We start with assuming that the outside options \(d_1 = (d_{11}, d_{12})\) and \(d_2 = (d_{21}, d_{22})\) satisfy \(d_{12} > d_{22}, d_{21} > d_{11}, d_{11} + d_{12} \leq 1, d_{21} + d_{22} \leq 1\) so that players prefer the other party to opt out instead of themselves to opt out. We further assume that if both leave the table simultaneously they get \(d_S = (d_{S1}, d_{S2})\).

**Proposition 2** Suppose that the payoffs to opting out, \(d_1\) and \(d_2\), satisfy \(d_{12} > d_{22}, d_{21} > d_{11}, d_{11} + d_{12} \leq 1, d_{21} + d_{22} \leq 1\), so that players prefer the other party to opt out instead of themselves to opt out. If \(\frac{d_{11}}{\delta_1} + \frac{d_{22}}{\delta_2} \leq 1\), \(d_{ii} \geq \delta_i (1 - \frac{d_{ii}}{\delta_i})\) for \(i, j \in \{1, 2\}\) with \(i \neq j\), then the bargaining game has at least one subgame perfect equilibrium strategy profile \((\phi_1^*, \phi_2^*)\) that result in immediate and efficient agreement. In particular, let \(\psi, \omega \in [\frac{d_{11}}{\delta_1}, 1 - \frac{d_{22}}{\delta_2}]\). Then, the
following strategies constitute a subgame perfect equilibrium where players agree upon $(ψ, 1 − ψ)$ in the initial period:

- **Player 1 always proposes** $(ψ, 1 − ψ)$, **rejects a proposal** $(ω, 1 − ω)$ **if and only if** $ω < ψ$, and **never opts out**;
- **Player 2 always proposes** $(ω, 1 − ω)$, **rejects a proposal** $(ψ, 1 − ψ)$ **if and only if** $ψ > ψ$, and **never opts out**.

**Proof.** CASE 1: Consider a subgame starting in an even period $t$.

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in agreed-upon shares $(ψ, 1 − ψ)$. Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer $(ψ, 1 − ψ)$ with $ψ ≤ ψ$ of Player 1 in some even period $i ≥ t$. As $ψ ≤ ψ$, this is not profitable;

2. After an accepted offer $(ω, 1 − ω)$ of Player 2 in some odd period $i > t$. As $ψ, ω ∈ [d_{11}/d_{11}, 1 − d_{22}/d_{22}]$, it follows that $δ_i ω ≤ ψ$, so this is not profitable;

3. In disagreement outcome $d_1$. As $d_{11} ≤ ψ$ by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in agreed-upon shares $(ψ, 1 − ψ)$. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer $(ψ, 1 − ψ)$ of Player 1 in some even period $i > t$. Clearly, deferring this payoff is not profitable;

2. After an accepted offer $(ω, 1 − ω)$ with $ω ≥ ω$ of Player 2 in some odd period $i > t$. As $ψ, ω ∈ [d_{11}/d_{11}, 1 − d_{22}/d_{22}]$, it follows that $δ_2(1 − ω) ≤ δ_2(1 − ω) ≤ 1 − ψ$, so this is not profitable;

3. In disagreement outcome $d_2$. As $d_{22} ≤ (1 − ψ)$ by assumption, this is not profitable.

CASE 2: Consider a subgame starting in an odd period $t$.

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in agreed-upon shares $(ω, 1 − ω)$. Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer $(ψ, 1 − ψ)$ with $ψ ≤ ψ$ of Player 1 in some even period $i > t$. As $ψ, ω ∈ [d_{11}/d_{11}, 1 − d_{22}/d_{22}]$, it follows that $δ_1 ψ ≤ ω$, so this is not profitable;
2. After an accepted offer \((\omega, 1 - \omega)\) of Player 2 in some odd period \(\bar{t} > t\). Clearly, deferring this payoff is not profitable;

3. In disagreement outcome \(d_1\). As \(d_{11} \leq \omega\) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in agreed-upon shares \((\omega, 1 - \omega)\). Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer \((\psi, 1 - \psi)\) of Player 1 in some even period \(\bar{t} > t\). As \(\psi, \omega \in \left[\frac{\delta_1 - \delta_1 \delta_2}{1 - \delta_1 \delta_2}, \frac{1 - \delta_2}{1 - \delta_1 \delta_2}\right]\), it follows that \(\delta_2(1 - \psi) \leq (1 - \omega)\), so this is not profitable;

2. After an accepted offer \((\bar{\omega}, 1 - \bar{\omega})\) with \(\bar{\omega} \geq \omega\) of Player 2 in some odd period \(\bar{t} \geq t\). As \(\bar{\omega} \leq \omega\), this is not profitable;

3. In disagreement outcome \(d_2\). As \(d_{22} \leq (1 - \omega)\) by assumption, this is not profitable.

We note that the strategies of the players don’t contain the option of opting out since opting out for the proposers is no longer credible; instead the players are locked in the game not by assumption as in Rubinstein [8] but as a consequence of the payoff structure. Hence, the main difference of the equilibria in proposition 1 and 2 from the immediate equilibria in the previous models with two-sided anonymous outside options is that agreement is not reached under the coercion of opting out since it is no longer intimidating, but under the time pressure. We also note that we have multiplicity of equilibria even in this simple setting.

**Proposition 3** In the bargaining game described in Proposition 2, there exist subgame-perfect equilibria leading to an immediate (and inefficient if \(d_{11} + d_{12} < 1, d_{21} + d_{22} < 1\)) outcome, where both players make nonserious offers to each other, and one of the players take his/her outside option, either \(d_1 = (d_{11}, d_{12})\) or \(d_2 = (d_{21}, d_{22})\). In particular, if \(\delta_2 d_{12} \leq d_{22}\) and \(\delta_1 d_{21} \leq d_{11}\), then the following strategies constitute a subgame perfect equilibrium:

i. The proposer always asks for 1 and never opts out, the responder rejects any offer less than 1 and opts out. Then the game has the immediate outcome \(d_2\).

ii. The proposer always asks for 1 and if the responder rejects, then the proposer opts out, the responder rejects any offer less than 1 and never opts out. Then the game has the immediate outcome \(d_1\).

**Proof.**

The proof for i.
CASE 1: Consider a subgame starting in an even period $t$.

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in disagreement outcome $d_2$. Suppose she deviates. Given Player 2's strategy, the game can end in only three cases:

1. After a rejected offer $(\hat{\psi}, 1 - \hat{\psi})$ with $0 < \hat{\psi}$ of Player 1 followed by opting out by Player 2 in some even period $\tilde{t} \geq t$. Since Player 2 will reject and opt out in any case, the outcome will still be $d_2$;

2. After an accepted offer $(0, 1)$ of Player 1 by Player 2 in some even period $\tilde{t} \geq t$. Clearly, this is not profitable;

3. In disagreement outcome $d_1$. As $d_{21} > d_{11}$ by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in disagreement outcome $d_2$. Suppose he deviates. Given Player 1's strategy, the game can end in only four cases:

1. After an accepted offer $(1, 0)$ of Player 1 in some even period $\tilde{t} \geq t$. Clearly, this is not profitable;

2. After an accepted offer $(1, 0)$ of Player 2 by Player 1 in some odd period $\tilde{t} > t$. Clearly, this is not profitable;

3. After a rejected offer $(\tilde{\omega}, 1 - \tilde{\omega})$ with $\tilde{\omega} < 1$ of Player 2 followed by opting out by Player 1 in some odd period $\tilde{t} > t$. As it yields the payoff $d_{12}$, this is not profitable since by assumption $d_{21} d_{12} \leq d_{22}$;

4. In disagreement outcome $d_2$ in some period $\tilde{t} > t$. Clearly, deferring this payoff is not profitable.

CASE 2: Consider a subgame starting in an odd period $t$.

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in disagreement outcome $d_1$. Suppose she deviates. Given Player 2’s strategy, the game can end in only four cases:

1. After a rejected offer $(\hat{\psi}, 1 - \hat{\psi})$ with $\hat{\psi} > 0$ of Player 1 followed by opting out by Player 2 in some even period $\tilde{t} > t$. As it yields the payoff $d_2$; this is not profitable since by assumption $d_1 d_{21} \leq d_{11}$;

2. After an accepted offer $(0, 1)$ of Player 1 by Player 2 in some even period $\tilde{t} > t$. Clearly, this is not profitable;
3. After an accepted offer \( (0, 1) \) of Player 2 in some odd period \( \tilde{t} \geq t \). Clearly, this is not profitable;

4. In disagreement outcome \( d_1 \) in some period \( \tilde{t} > t \). Clearly, deferring this payoff is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in disagreement outcome \( d_1 \). Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After a rejected offer \( (\tilde{\omega}, 1 - \tilde{\omega}) \) with \( \tilde{\omega} < 1 \) of Player 2 followed by opting out by Player 1 in some odd period \( \tilde{t} \geq t \). Since Player 1 will reject and opt out in any case, the outcome will still be \( d_1 \);

2. After an accepted offer \( (1, 0) \) of Player 2 in some odd period \( \tilde{t} \geq t \). Clearly, this is not profitable;

3. In disagreement outcome \( d_2 \). As \( d_{22} < d_{12} \) by assumption, this is not profitable.

**The Proof for \( \bar{u} \).**

**Case 1:** Consider a subgame starting in an even period \( t \).

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in disagreement outcome \( d_1 \). Suppose she deviates. Given Player 2’s strategy, the game can end in only four cases:

1. After an accepted offer \( (0, 1) \) of Player 2 in some odd period \( \tilde{t} > t \). Clearly, this is not profitable;

2. After an accepted offer \( (0, 1) \) of Player 1 by Player 2 in some even period \( \tilde{t} \geq t \). Clearly, this is not profitable;

3. After a rejected offer \( (\tilde{\omega}, 1 - \tilde{\omega}) \) with \( \tilde{\omega} < 1 \) of Player 2 by Player 1 followed by opting out by Player 2 in some odd period \( \tilde{t} > t \). As it yields the payoff \( d_2 \), this is not profitable since by assumption \( \delta_{12} d_{21} \leq d_{11} \);

4. In disagreement outcome \( d_1 \) in some period \( \tilde{t} > t \). Clearly, deferring this payoff is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in disagreement outcome \( d_1 \). Suppose he deviates. Given Player 1’s strategy, the game can end in only two cases:
1. After an accepted offer \((1, 0)\) of Player 1 in some even period \(\hat{t} \geq t\). Clearly, this is not profitable;

2. In disagreement outcome \(d_2\). As \(d_{22} < d_{12}\) by assumption, this is not profitable.

**Case 2:** Consider a subgame starting in an odd period \(t\).

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in disagreement outcome \(d_2\). Suppose she deviates. Given Player 2’s strategy, the game can end in only two cases:

1. After an accepted offer \((0, 1)\) of Player 2 in some odd period \(\hat{t} \geq t\). Clearly, this is not profitable;

2. In disagreement outcome \(d_1\). As \(d_{11} < d_{21}\) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in disagreement outcome \(d_2\). Suppose he deviates. Given Player 1’s strategy, the game can end in only four cases:

1. After an accepted offer \((1, 0)\) of Player 1 in some even period \(\hat{t} > t\). Clearly, this is not profitable;

2. After an accepted offer \((1, 0)\) of Player 2 in some odd period \(\hat{t} \geq t\). Clearly, this is not profitable;

3. After a rejected offer \((1, 0)\) of Player 1 by Player 2 followed by opting out by Player 1 in some even period \(\hat{t} > t\). As it yields the payoff \(d_1\), this is not profitable since by assumption \(\delta_2d_{12} \leq d_{22}\);

4. In disagreement outcome \(d_2\) in some period \(\hat{t} > t\). Clearly, deferring this payoff is not profitable.

---

Proposition 3 indicates that, when waiting is not so attractive, instead of locking in the game, players can choose to stick to nonserious offers, forcing the other to opt out.

The following proposition is a modification of the theorem in Ponsatí and Sákovics [7]. The delayed strategies are constructed such as the game transits to a final state after a series of nonserious offers stage, and the last state which is never reached is just for the sake of truncating the game (i.e. avoiding the nonagreement case, since it yields payoffs no better than the previous state). Again, the equilibria is attained without the threat of opting out, yet some equilibria are delayed.
Proposition 4 If \( \delta_i^2(1 - \frac{d_{ij}}{\delta_j}) \leq d_{ii} \leq \delta_i(1 - \frac{d_{ij}}{\delta_j}) \) for \( i, j \in \{1, 2\} \) with \( i \neq j \), then the bargaining game has subgame perfect equilibria that result in immediate and efficient agreement. In particular, let \( \psi \in [\delta_1(1 - \frac{d_{22}}{\delta_2}), 1 - d_{22}] \):

i. the following strategies written as automata\(^5\) in Table 1\(^6\) constitute a subgame perfect equilibrium where players agree upon \((\psi, 1 - \psi)\) in the initial period:

<p>| Table 1. The equilibrium strategies for immediate equilibria |
|---------------------------------|-----------------|</p>
<table>
<thead>
<tr>
<th></th>
<th>Base state</th>
<th>Threat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>proposes ((\psi, 1 - \psi))</td>
<td>((\frac{d_{11}}{\delta_1}, 1 - \frac{d_{11}}{\delta_1}))</td>
</tr>
<tr>
<td></td>
<td>accepts -</td>
<td>(\tilde{\omega} \geq 1 - \frac{d_{22}}{\delta_2})</td>
</tr>
<tr>
<td></td>
<td>opts out no</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>proposes -</td>
<td>((1 - \frac{d_{22}}{\delta_2}, \frac{d_{22}}{\delta_2}))</td>
</tr>
<tr>
<td></td>
<td>accepts (\tilde{\psi} \geq 1 - \psi)</td>
<td>(\tilde{\psi} \geq 1 - \frac{d_{11}}{\delta_1})</td>
</tr>
<tr>
<td></td>
<td>opts out no</td>
<td>no</td>
</tr>
<tr>
<td>Transitions</td>
<td>Go to Threat if Player 2 rejects</td>
<td>Absorbing</td>
</tr>
</tbody>
</table>

Furthermore, if \( \delta_i^2(1 - \frac{d_{ij}}{\delta_j}) \leq d_{ii} \leq \delta_i(1 - \frac{d_{ij}}{\delta_j}) \) for \( i, j \in \{1, 2\} \) with \( i \neq j \), then the bargaining game has subgame perfect equilibria with delay that result in efficient agreement at the time of the agreement. In particular, let \( \psi \in [\delta_1^{-t}(1 - \frac{d_{22}}{\delta_2}), 1 - \delta_1^{-t}(1 - \delta_1^2(1 - \frac{d_{22}}{\delta_2}))] \), whenever such \( \psi \) exist;

ii. the following strategies written as automata in Table 2 constitute a subgame perfect equilibrium where players agree upon \((\psi, 1 - \psi)\) in the initial period:

<p>| Table 2. The equilibrium strategies for delayed equilibria |
|---------------------------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th></th>
<th>Nonserious offers state</th>
<th>Base state</th>
<th>Threat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>proposes ((1, 0))</td>
<td>((\psi, 1 - \psi))</td>
<td>((\frac{d_{11}}{\delta_1}, 1 - \frac{d_{11}}{\delta_1}))</td>
</tr>
<tr>
<td></td>
<td>accepts (\tilde{\omega} \geq 1)</td>
<td>-</td>
<td>(\tilde{\omega} \geq 1 - \frac{d_{22}}{\delta_2})</td>
</tr>
<tr>
<td></td>
<td>opts out no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>proposes ((0, 1))</td>
<td>-</td>
<td>((1 - \frac{d_{22}}{\delta_2}, \frac{d_{22}}{\delta_2}))</td>
</tr>
<tr>
<td></td>
<td>accepts (\tilde{\psi} \geq 1)</td>
<td>(\tilde{\psi} \geq 1 - \psi)</td>
<td>(\tilde{\psi} \geq 1 - \frac{d_{11}}{\delta_1})</td>
</tr>
<tr>
<td></td>
<td>opts out no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Transitions</td>
<td>Go to Base state after (t-1) odd periods</td>
<td>Go to Threat if Player 2 rejects</td>
<td>Absorbing</td>
</tr>
</tbody>
</table>


\(^6\)Table 1 reads as follows: The game starts in the Base State with an offer \((\psi, 1 - \psi)\) by Player 1. Player 2 accepts any payoff \(z \geq 1 - \psi\) offered to him. Both players don’t choose to opt out. In case Player 2 rejects her offer, the state transits to 'Threat' and players continue to play the (sub)game with the prescribed strategies under the state 'Threat'. The dashes in Table 1 refer to those subgames which can never be reached. An 'Absorbing' state simply means that, once it is reached, the game either ends or continues in that state. Table 2 and Table 3 also reads analogously as Table 1 does.
Proof.

The Proof for $t$.

Case 1: Consider a subgame starting in an even period $t$ in ‘Base State’.

Player 1 plays a best response: Sticking to her current strategy, the subgame ends immediately in agreed-upon shares $(\psi, 1 - \psi)$. Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer $(\tilde{\psi}, 1 - \tilde{\psi})$ with $\tilde{\psi} \leq \frac{d_{11}}{\delta_1}$ of Player 1 in some even period $\tilde{t} > t$ in the ‘Threat State’. As $\psi \in [\delta_1(1 - \frac{d_{22}}{\delta_2}), 1 - d_{22}]$, so that $\delta_1 \tilde{\psi} \leq d_{11} \leq \psi$, this is not profitable;

2. After an accepted offer $(1 - \frac{d_{22}}{\delta_2}, \frac{d_{21}}{\delta_2})$ of Player 2 in some odd period $\tilde{t} > t$ in the ‘Threat State.’ As $\psi \in [\delta_1(1 - \frac{d_{22}}{\delta_2}), 1 - d_{22}]$, it follows that $\delta_1 \left(1 - \frac{d_{22}}{\delta_2}\right) \leq \psi$, so this is not profitable;

3. In disagreement outcome $d_1$. As $d_{11} \leq \psi$ by assumption, this is not profitable.

Player 2 plays a best response: Sticking to his current strategy, the subgame ends immediately in agreed-upon shares $(\psi, 1 - \psi)$. Note that the game cannot end after an accepted offer $(\psi, 1 - \psi)$ of Player 1 or $(\omega, 1 - \omega)$ of Player 2 in some even period $\hat{t} > t$ in the ‘Base State’ since the subgames that end with such outcomes are never reached. So, we rule out those cases. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer $(\frac{d_{11}}{\delta_1}, 1 - \frac{d_{11}}{\delta_1})$ of Player 1 by Player 2 in some even period $\hat{t} > t$ in the ‘Threat State.’ As $\psi \in [\delta_1(1 - \frac{d_{22}}{\delta_2}), 1 - d_{22}]$, it follows that $\delta_2 \left(1 - \frac{d_{11}}{\delta_1}\right) \leq 1 - \psi$, so this is not profitable;

2. After an accepted offer $(\tilde{\omega}, 1 - \tilde{\omega})$ with $\tilde{\omega} \geq 1 - \frac{d_{22}}{\delta_2}$ of Player 2 in some odd period $\hat{t} > t$ in the ‘Threat State.’ As $\psi \in [\delta_1(1 - \frac{d_{22}}{\delta_2}), 1 - d_{22}]$, it follows that $\delta_2 (1 - \tilde{\omega}) \leq \delta_2 (1 - (1 - \frac{d_{22}}{\delta_2})) \leq 1 - \psi$, so this is not profitable;

3. In disagreement outcome $d_2$. As $d_{22} \leq (1 - \psi)$ by assumption, this is not profitable.

Take the subgame starting at period $t$ in the ‘Base State’ and suppose that Player 2 offers $(\omega, 1 - \omega)$. Note that the game cannot end after an accepted offer $(\psi, 1 - \psi)$ of Player 1 or $(\omega, 1 - \omega)$ of Player 2 in some even period $\hat{t} > t$ in the ‘Base State’, and hence the subgames that end with such outcomes are never reached, since, given that Player 1 starts to offer $(\frac{d_{11}}{\delta_1}, 1 - \frac{d_{11}}{\delta_1})$ and accepts any offer $\tilde{\omega} \geq 1 - \frac{d_{22}}{\delta_2}$, sticking the prescribed equilibrium strategy for Player 2 is optimal, and the game transits to the ‘Threat Stage,’ which we show below:
CASE 2: Consider a subgame starting in an odd period $t$ in the ‘Threat State.’

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in agreed-upon shares $(1 - \frac{d_{22}}{\delta_2}, \frac{d_{22}}{\delta_2})$. Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer $(\tilde{\psi}, 1 - \tilde{\psi})$ with $\tilde{\psi} \leq \frac{d_{11}}{\delta_1}$ of Player 1 in some even period $\tilde{t} > t$.
   This is not profitable since $\delta_1 \tilde{\psi} \leq d_{11} \leq \delta_1(1 - \frac{d_{22}}{\delta_2}) \leq (1 - \frac{d_{22}}{\delta_2})$ by assumption;

2. After an accepted offer $(1 - \frac{d_{22}}{\delta_2}, \frac{d_{22}}{\delta_2})$ of Player 2 in some odd period $\tilde{t} > t$ in the ‘Threat State.’ Clearly, deferring this payoff is not profitable;

3. In disagreement outcome $d_1$. As $d_{11} \leq \delta_1(1 - \frac{d_{22}}{\delta_2}) \leq 1 - \frac{d_{22}}{\delta_2}$ by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in agreed-upon shares $(1 - \frac{d_{22}}{\delta_2}, \frac{d_{22}}{\delta_2})$. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer $(\tilde{\omega}, 1 - \tilde{\omega})$ with $\tilde{\omega} \geq 1 - \frac{d_{22}}{\delta_2}$ of Player 2 in some odd period $\tilde{t} > t$.
   As $\tilde{\omega} \leq 1 - \frac{d_{22}}{\delta_2}$, this is not profitable;

2. After an accepted offer $(\tilde{\omega}, 1 - \tilde{\omega})$ with $\tilde{\omega} = 1 - \frac{d_{22}}{\delta_2}$ of Player 2 in some even period $\tilde{t} > t$.
   As $\tilde{\omega} \leq 1 - \frac{d_{22}}{\delta_2}$, this is not profitable;

3. In disagreement outcome $d_2$. As $d_{22} \leq \frac{d_{22}}{\delta_2}$, this is not profitable.

CASE 3: Consider a subgame starting in an even period $t$ in ‘Threat State.’

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in agreed-upon shares $(\frac{d_{11}}{\delta_1}, 1 - \frac{d_{11}}{\delta_1})$. Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer $(\tilde{\psi}, 1 - \tilde{\psi})$ with $\tilde{\psi} \leq \frac{d_{11}}{\delta_1}$ of Player 1 in some even period $\tilde{t} > t$.
   As $\tilde{\psi} \leq \frac{d_{11}}{\delta_1}$, this is not profitable;

2. After an accepted offer $(1 - \frac{d_{22}}{\delta_2}, \frac{d_{22}}{\delta_2})$ of Player 2 in some odd period $\tilde{t} > t$.
   As $\frac{d_{11}}{\delta_1} \geq \delta_1(1 - \frac{d_{22}}{\delta_2})$ by assumption, this is not profitable;

3. In disagreement outcome $d_1$. As $d_{11} \leq \frac{d_{11}}{\delta_1}$, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in agreed-upon shares $(\frac{d_{11}}{\delta_1}, 1 - \frac{d_{11}}{\delta_1})$. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:
1. After an accepted offer \((d_{11} \delta_{1}, 1 - d_{11} \delta_{1})\) of Player 1 in some even period \(\tilde{t} > t\). Clearly, deferring this payoff is not profitable;

2. After an accepted offer \((\tilde{\omega}, 1 - \tilde{\omega})\) with \(\tilde{\omega} \geq 1 - d_{22} \delta_{2}\) of Player 2 in some odd period \(\tilde{t} > t\). As \(1 - \tilde{\omega} \leq d_{22} \delta_{2}\), this is not profitable;

3. In disagreement outcome \(d_{2}\). As \(d_{22} \leq \delta_{2}^{3}(1 - d_{11} \delta_{1}) \leq 1 - d_{11} \delta_{1}\) by assumption, this is not profitable.

**THE PROOF FOR ii.**

**CASE 1:** Consider a subgame starting in an even period \(s\) in the ‘Nonserious Offers State.’

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends in agreed-upon shares \((\psi, 1 - \psi)\) at period \(t\), when the ‘Base State’ is reached. Suppose she deviates. Given Player 2’s strategy, the game can end in only five cases:

1. After an accepted offer \((0, 1)\) of Player 2 in some odd period \(s < \bar{s} < t\), which is definitely not profitable;

2. After an accepted offer \((0, 1)\) of Player 1 by Player 2 in some even period \(s \leq \bar{s} < t\), which is definitely not profitable;

3. After an accepted offer \((1 - d_{22} \delta_{2}, d_{22} \delta_{2})\) of Player 2 in some odd period \(\tilde{t} > t\) in the ‘Threat State.’ As \(\psi \in [\delta_{1}^{1-t}(1 - d_{22} \delta_{2}), 1 - \delta_{2}^{1-t}(1 - \delta_{2}^{2}(1 - d_{22} \delta_{2}))]\), it follows that \(\delta_{1}^{1-t}(1 - d_{22} \delta_{2}) \leq \psi\), so this is not profitable;

4. After an accepted offer \((\tilde{\psi}, 1 - \tilde{\psi})\) with \(\tilde{\psi} \leq d_{11} \delta_{1}\) of Player 1 in some even period \(\tilde{t} > t\) in the ‘Threat State.’ As \(\psi \in [\delta_{1}^{1-t}(1 - d_{22} \delta_{2}), 1 - \delta_{2}^{1-t}(1 - \delta_{2}^{2}(1 - d_{22} \delta_{2}))]\), it follows that that \(\delta_{1}^{1-t}(1 - d_{22} \delta_{2}) \leq \psi\), this is not profitable;

5. In disagreement outcome \(d_{1}\). As \(d_{11} \leq \delta_{1}(1 - d_{22} \delta_{2}) \leq \delta_{1}^{1-t}\delta_{1}^{1-t}(1 - d_{22} \delta_{2}) \leq \delta_{1}^{1-t}(1 - d_{22} \delta_{2})\) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends in agreed-upon shares \((\psi, 1 - \psi)\) at period \(t\), when the ‘Base State’ is reached. Suppose he deviates. Given Player 1’s strategy, the game can end in only five cases:

1. After an accepted offer \((1, 0)\) of Player 1 in some even period \(s \leq \bar{s} < t\), which is definitely not profitable;

2. After an accepted offer \((1, 0)\) of Player 2 by Player 1 in some odd period \(s < \bar{s} < t\), which is definitely not profitable;
3. After an accepted offer \((\hat{\omega}, 1 - \hat{\omega})\) with \(\hat{\omega} \geq 1 - \frac{d_{22}}{\delta_2}\) of Player 2 in some odd period \(\tilde{t} > t\) in the ‘Threat State.’ As \(\psi \in [\delta_1^{-t}(1 - \frac{d_{22}}{\delta_2}), 1 - \delta_2^{-t}(1 - \delta_1^{t}(1 - \frac{d_{22}}{\delta_2}))\], and using the inequality \((1 - \delta_2^{t}\frac{d_{22}}{\delta_2}) \leq \delta_1^{t}(1 - \frac{d_{22}}{\delta_2})\), it follows that \(\delta_2^{t}\frac{d_{22}}{\delta_2} = d_{22} \leq 1 - \psi\), so this is not profitable;

4. After an accepted offer \((\psi, 1 - \psi)\) of Player 1 in some even period \(\tilde{t} > t\) in the ‘Threat State.’ Clearly, deferring this payoff is not profitable;

5. In disagreement outcome \(d_2\). As \(\psi \in [\delta_1^{-t}(1 - \frac{d_{22}}{\delta_2}), 1 - \delta_2^{-t}(1 - \delta_1^{t}(1 - \frac{d_{22}}{\delta_2}))\], and using the inequality \((1 - \delta_2^{t}\frac{d_{22}}{\delta_2}) \leq \delta_1^{t}(1 - \frac{d_{22}}{\delta_2})\), it follows that \(d_{22} \leq \delta_2^{t-s}(1 - \psi)\), so this is not profitable.

**Case 2:** Consider a subgame starting in an odd period \(s\) in the ‘Nonserious Offers State.’

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends in agreed-upon shares \((\psi, 1 - \psi)\) at period \(t\), when the ‘Base State’ is reached. Suppose she deviates. Given Player 2’s strategy, the game can end in only five cases:

1. After an accepted offer \((0, 1)\) of Player 2 in some odd period \(s < \tilde{s} < t\), which is definitely not profitable;

2. After an accepted offer \((0, 1)\) of Player 1 by Player 2 in some even period \(s < \tilde{s} < t\), which is definitely not profitable;

3. After an accepted offer \((1 - \frac{d_{22}}{\delta_2}, \frac{d_{22}}{\delta_2})\) of Player 2 in some odd period \(\tilde{t} > t\) in the ‘Threat State.’ As \(\psi \in [\delta_1^{-t}(1 - \frac{d_{22}}{\delta_2}), 1 - \delta_2^{-t}(1 - \delta_1^{t}(1 - \frac{d_{22}}{\delta_2}))\], it follows that \(\delta_1\left(1 - \frac{d_{22}}{\delta_2}\right) \leq \psi\), so this is not profitable;

4. After an accepted offer \((\hat{\psi}, 1 - \hat{\psi})\) with \(\hat{\psi} \leq \frac{d_{11}}{\delta_1}\) of Player 1 in some even period \(\tilde{t} > t\) in the ‘Threat State,’ As \(\psi \in [\delta_1^{-t}(1 - \frac{d_{22}}{\delta_2}), 1 - \delta_2^{-t}(1 - \delta_1^{t}(1 - \frac{d_{22}}{\delta_2}))\], it follows that that \(\delta_1\hat{\psi} \leq d_{11} \leq \psi\), this is not profitable;

5. In disagreement outcome \(d_1\). As \(d_{11} \leq \delta_1(1 - \frac{d_{22}}{\delta_2}) \leq \delta_1^{t-s}\delta_1^{-t}(1 - \frac{d_{22}}{\delta_2}) \leq \delta_1^{t-s}\psi\) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends in agreed-upon shares \((\psi, 1 - \psi)\) at period \(t\), when the ‘Base State’ is reached. Suppose he deviates. Given Player 1’s strategy, the game can end in only five cases:

1. After an accepted offer \((1, 0)\) of Player 1 in some even period \(s \leq \tilde{s} < t\), which is definitely not profitable;
2. After an accepted offer \((1, 0)\) of Player 2 by Player 1 in some odd period \(s \leq \tilde{s} < t\), which is definitely not profitable;

3. After an accepted offer \((\tilde{\omega}, 1 - \tilde{\omega})\) with \(\tilde{\omega} \geq 1 - \tfrac{d_{22}}{\delta_2}\) of Player 2 in some odd period \(\tilde{t} > t\) in the ‘Threat State.’ As \(\psi \in [\delta_1^{-t}(1 - \tfrac{d_{22}}{\delta_2}), 1 - \delta_2^{1-t}(1 - \delta_1^2(1 - \tfrac{d_{22}}{\delta_2}))]\), it follows that 
\[\delta_2 \frac{d_{22}}{\delta_2} = d_{22} \leq 1 - \psi,\] so this is not profitable;

4. After an accepted offer \((\psi, 1 - \psi)\) of Player 1 in some even period \(\tilde{t} > t\) in the ‘Threat State.’ Clearly, deferring this payoff is not profitable;

5. In disagreement outcome \(d_2\). As \(d_{22} \leq \delta_2^{l-s}(1 - \psi)\) by assumption, this is not profitable.

There are three other cases, specifically the subgames starting in an even period \(t\) in ‘Base State,’ in an odd period \(t\) in ‘Threat State,’ and in an even period \(t\) in ‘Threat State,’ which can be treated similarly as in the proof of part 4. So, we skip those.

Note that the strategies for equilibria with delay as shown in Table 2 are derived from the strategies in the previous case by adding a new state in which nonserious offers are made until the agreement date \(t\), the last nonserious offer is made by Player 2 at date \(t - 1\) and then the state transits to the ‘Base State’ in which Player 1 offers.

Observe that the strategies of the players don’t contain the option of opting out since opting out for the proposers is no longer credible; instead the players are locked in the game not by assumption as in Rubinstein [8] but as a consequence of the payoff structure. Hence, the main difference of the equilibria in Proposition 1 and 2 from the immediate equilibria in the previous models with two-sided anonymous outside options is that agreement is not reached under the coercion of opting out since it is no longer intimidating, but under the time pressure. We also note that we have multiplicity of equilibria even in this simple setting, and that when the outside options approach to 0, the equilibria reported above will no longer be equilibria, contrary to the result in Ponsati and Sákovics [7].

Rubinstein [8] shows the uniqueness of the equilibrium when it is assumed that players are locked in the game. Ponsati and Sákovics [7] show that there is a continuum of equilibria (even some with delay) when players are allowed to quit at any time despite their outside options are zero. We here show that the game may admit a continuum of equilibria (even some with delay) though the players are locked (or compelled to stay) in the game, not of course by assumption, but as a consequence of the payoff structure. If such an asymmetric payoff structure (of outside options) is insitutionally set to attain immediate and efficient equilibria, we can conclude that forcing the bargaining parties to stay around the table may not guarantee immediate and efficient equilibria.
Furthermore, obstinacy may cause severe delays with disagreement. If the players’ own disagreement payoffs are zero, one can construct equilibria with an arbitrary length of delay. The following proposition constitutes such an example:

**Proposition 5** If the players’ own disagreement payoffs are zero, i.e. \( d_1 = (0, d_{12}) \) and \( d_2 = (d_{21}, 0) \) with \( d_{12}, d_{21} > 0 \), there exist equilibria with significant delay, where players go into a series of nonserious offers, followed by surrender of one of them at an arbitrary finite period \( t \). In particular, the following strategies written as automata in Table 3 constitute a subgame perfect equilibrium where the game ends with the disagreement outcome \( d_2 \) in period \( t \):

<table>
<thead>
<tr>
<th></th>
<th>Nonserious offers state</th>
<th>Base state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>proposes (1, 0)</td>
<td>(1, 0)</td>
</tr>
<tr>
<td></td>
<td>accepts ( \psi \geq 1 )</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>opts out no</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>proposes (0, 1)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>accepts ( \tilde{\omega} \geq 1 )</td>
<td>( \tilde{\omega} \geq 1 )</td>
</tr>
<tr>
<td></td>
<td>opts out no</td>
<td>yes</td>
</tr>
</tbody>
</table>

Transitions: Go to Base state after \( t-1 \) odd periods of play if Player 2 deviates

### Proof.

**Case 1:** Consider a subgame starting in an even period \( s \) in the ‘Nonserious Offers State.’

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends in disagreement outcome \( d_2 \) at period \( t \), when the ‘Base State’ is reached. Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer \( (0, 1) \) of Player 2, which is definitely not profitable;
2. After an accepted offer \( (0, 1) \) of Player 1 by Player 2, which is definitely not profitable;
3. In disagreement outcome \( d_1 \). As \( d_{11} = 0 \) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends in disagreement outcome \( d_2 \) at period \( t \), when the ‘Base State’ is reached. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer \( (1, 0) \) of Player 1, which is definitely not profitable;
2. After an accepted offer \((1, 0)\) of Player 2 by Player 1, which is definitely not profitable;

3. In disagreement outcome \(d_2\). As \(d_{22} = 0\) by assumption, this is not profitable.

**CASE 2:** Consider a subgame starting in an odd period \(s\) in the ‘Nonserious Offers State.’

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends in disagreement outcome \(d_2\) at period \(t\), when the ‘Base State’ is reached. Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer \((0, 1)\) of Player 2, which is definitely not profitable;

2. After an accepted offer \((0, 1)\) of Player 1 by Player 2, which is definitely not profitable;

3. In disagreement outcome \(d_1\). As \(d_{11} = 0\) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends in disagreement outcome \(d_2\) at period \(t\), when the ‘Base State’ is reached. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer \((1, 0)\) of Player 1, which is definitely not profitable;

2. After an accepted offer \((1, 0)\) of Player 2 by Player 1, which is definitely not profitable;

3. In disagreement outcome \(d_2\). As \(d_{22} = 0\) by assumption, this is not profitable.

Take the subgame starting at period \(t\) in the ‘Base State’ and suppose that Player 2 offers \((\omega, 1 - \omega)\). Note that the game cannot end after an accepted offer \((\psi, 1 - \psi)\) of Player 1 or \((\omega, 1 - \omega)\) of Player 2 in some even period \(i > t\) in the ‘Base State’, and hence the subgames that end with such outcomes are never reached, since, given that Player 1 starts to offer \((1, 0)\) and accepts any offer \(\omega \geq 1\), sticking the prescribed equilibrium strategy for Player 2 is optimal, and the game transits to the ‘Nonserious Offers Stage’ back again, which we showed above. As a result, we will not consider such subgames.

**CASE 3:** Consider a subgame starting in an even period \(s\) in the ‘Base State.’

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in disagreement outcome \(d_2\) at period \(s\), when the ‘Base State’ is reached. Suppose she deviates. Given Player 2’s strategy, the game can end in only three cases:

1. After an accepted offer \((0, 1)\) of Player 2, which is definitely not profitable;

2. After an accepted offer \((0, 1)\) of Player 1 by Player 2, which is definitely not profitable;
3. In disagreement outcome $d_1$. As $d_{11} = 0$ by assumption, this is not profitable.

Player 2 plays a best response: Sticking to his current strategy, the subgame ends in disagreement outcome $d_2$ at period $s$, when the ‘Base State’ is reached. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer $(1, 0)$ of Player 1, which is definitely not profitable;
2. After an accepted offer $(1, 0)$ of Player 2 by Player 1, which is definitely not profitable;
3. In disagreement outcome $d_2$. As $d_{22} = 0$ by assumption, this is not profitable.

3.2 The case where one of the outside options is preferred by both players

Next, we will assume that one of the outside options, either $d_1 = (d_{11}, d_{12})$ or $d_2 = (d_{21}, d_{22})$ is preferred by both players. So, there are two cases to handle: one with $d_1 > d_2$, and the other with $d_1 < d_2$. We will just consider the case where $d_1 < d_2$ since the results for the other case is symmetric. Again, we assume that if both leave the table simultaneously they get $(\min\{d_{11}, d_{21}\}, \min\{d_{12}, d_{22}\}) = d_1$.

**Proposition 6** If $d_1 < d_2$, $1 - d_{21} \geq \frac{d_{22}}{d_2}$ then the bargaining game has at least one subgame perfect equilibrium that result in immediate and efficient agreement. In particular, let $\psi, \omega \in [1 - \frac{d_{22}}{d_2}, 1 - d_{22}]$. Then, the following strategies constitute a subgame perfect equilibrium where players agree upon $(\psi, 1 - \psi)$ in the initial period:

- **Player 1** always proposes $(\psi, 1 - \psi)$, rejects a proposal $(\tilde{\omega}, 1 - \tilde{\omega})$ if and only if $\tilde{\omega} < \omega$, and never opts out;
- **Player 2** always proposes $(\omega, 1 - \omega)$, opts out if his proposal is turned down, rejects a proposal $(\tilde{\psi}, 1 - \tilde{\psi})$ if and only if $\tilde{\psi} > \psi$ and opts out.

**Proof.**

CASE 1: Consider a subgame starting in an even period $t$.

Player 1 plays a best response: Sticking to her current strategy, the subgame ends immediately in agreed-upon shares $(\psi, 1 - \psi)$. Suppose she deviates. Given Player 2’s strategy, the game can end in only two cases:

1. In disagreement outcome $d_2$. As $d_{21} \leq \psi$ by assumption, this is not profitable;
2. In disagreement outcome $d_1$. As $d_{11} < d_{21} \leq \psi$ by assumption, this is not profitable.
**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in agreed-upon shares \((\psi, 1-\psi)\). Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer \((\psi, 1-\psi)\) of Player 1 in some even period \(\tilde{t} > t\). Clearly, deferring this payoff is not profitable;
2. After an accepted offer \((\tilde{\omega}, 1-\tilde{\omega})\) with \(\tilde{\omega} \geq \omega\) of Player 2 in some odd period \(\tilde{t} > t\). As \(\psi, \omega \in [1 - \frac{d_{22}}{d_2}, 1 - d_{22}]\), it follows that \(\delta_2(1-\tilde{\omega}) \leq \delta_2(1-\omega) \leq 1-\psi\), so this is not profitable;
3. In disagreement outcome \(d_2\). As \(d_{22} \leq (1-\psi)\) by assumption, this is not profitable.

**CASE 2:** Consider a subgame starting in an odd period \(t\).

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in agreed-upon shares \((\omega, 1-\omega)\). Suppose she deviates. Given Player 2’s strategy, the game can end in only two cases:

1. In disagreement outcome \(d_2\). As \(d_{21} \leq \omega\) by assumption, this is not profitable;
2. In disagreement outcome \(d_1\). As \(d_{11} < d_{21} \leq \omega\) by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in agreed-upon shares \((\omega, 1-\omega)\). Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer \((\psi, 1-\psi)\) of Player 1 in some even period \(\tilde{t} > t\). As \(\psi, \omega \in [1 - \frac{d_{22}}{d_2}, 1 - d_{22}]\), it follows that \(\delta_2(1-\psi) \leq (1-\omega)\), so this is not profitable;
2. After an accepted offer \((\tilde{\omega}, 1-\tilde{\omega})\) with \(\tilde{\omega} \geq \omega\) of Player 2 in some odd period \(\tilde{t} > t\). As \(1 - \tilde{\omega} \leq 1-\omega\), this is not profitable;
3. In disagreement outcome \(d_2\). As \(d_{22} \leq (1-\omega)\) by assumption, this is not profitable.

We also note that one can easily construct equilibria for the case when \(d_1 > d_2\) similar to those defined in proposition 6. Next, under the condition \(d_1 < d_2\), we write equilibria with nonserious offers that are similar to those in proposition 3.

**Proposition 7** If \(d_1 < d_2\), there exist subgame-perfect equilibria leading to an immediate (and inefficient if \(d_{21} + d_{22} < 1\)) disagreement outcome \(d_2 = (d_{21}, d_{22})\), where both players make nonserious offers to each other. In particular, the following strategies constitute a subgame perfect equilibrium:
Players always ask for 1, reject any offer less than 1, Player 1 never opts out, Player 2 opts out after she rejects a proposal, and in case Player 1 rejects his proposal. Then, the game has the immediate outcome $d_2 = (d_{21}, d_{22})$.

**Proof.**

**CASE 1:** Consider a subgame starting in an even period $t$.

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in the disagreement outcome $d_2$. Suppose she deviates. Given Player 2’s strategy, the game can end in only two cases:

1. After an accepted offer $(0, 1)$ of Player 1 by Player 2 in some even period $\tilde{t} \geq t$. This is clearly not profitable;
2. In disagreement outcome $d_1$. As $d_{11} < d_{21}$ by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in the disagreement outcome $d_2$. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:

1. After an accepted offer $(1, 0)$ of Player 1 in some even period $\tilde{t} \geq t$. This is clearly not profitable;
2. After an accepted offer $(1, 0)$ of Player 2 by Player 1 in some odd period $\tilde{t} > t$. This is not profitable since $d_{22} \geq 0$;
3. In disagreement outcome $d_2$ in some period $\tilde{t} > t$. Clearly, deferring this payoff is not profitable.

**CASE 2:** Consider a subgame starting in an odd period $t$.

**Player 1 plays a best response:** Sticking to her current strategy, the subgame ends immediately in the disagreement outcome $d_2$. Suppose she deviates. Given Player 2’s strategy, the game can end in only two cases:

1. After an accepted offer $(0, 1)$ of Player 2 in some odd period $\tilde{t} \geq t$. This is clearly not profitable;
2. In disagreement outcome $d_1$. As $d_{11} < d_{21}$ by assumption, this is not profitable.

**Player 2 plays a best response:** Sticking to his current strategy, the subgame ends immediately in the disagreement outcome $d_2$. Suppose he deviates. Given Player 1’s strategy, the game can end in only three cases:
1. After an accepted offer \((1, 0)\) of Player 1 in some even period \(i > t\). This is clearly not profitable;

2. After an accepted offer \((1, 0)\) of Player 2 by Player 1 in some odd period \(i \geq t\). This is clearly not profitable;

3. In disagreement outcome \(d_2\) in some period \(i > t\). Clearly, deferring this payoff is not profitable.

4 Conclusion

In the standard noncooperative bargaining game, Rubinstein [8] shows the uniqueness of the equilibrium when it is assumed that players are locked in the game. Ponsati and Sákovics [7] show that there is a continuum of equilibria (even some with delay) when players are allowed to quit at any time despite their outside options are zero. We here showed that when each player prefers the outside option of the other, the game may admit a continuum of equilibria (even some with delay) though the players are locked in the game, not of course by assumption, but as a consequence of the payoff structure. The interpretation of this is that if the players are compelled to stay in the game, e.g. by some institutional regulation such as the example of US labour dispute regulation given in the introduction, there may not be a unique immediate efficient outcome. So, we say that institutional regulation or societal pressure to keep bargaining parties around the table may not lead to unique, immediate, and efficient outcome; instead it may cause inefficiencies or significant delay in reaching an agreement.

Moreover, we have documented other equilibria under different payoff structures and showed in general that the extension of the standard alternating offers bargaining game with nonanonymous outside options enriches the set of equilibria.

References


