A Welfare Analysis of Pension Funds under Endogenous Labor Supply

Roel Mehlkopf

Tilburg University and Netspar

Abstract

Bodie, Merton and Samuelson (1992) have shown that the labor-leisure choice plays an influential role in intertemporal saving and investment decision making over the life cycle of an individual. Labor supply flexibility a blessing in their analysis because endogenous labor earnings provide a buffer against income shocks, thereby increasing the individual’s appetite for risk taking and improving ex ante welfare levels. This paper finds that the merits of labor supply flexibility are substantially reduced if the individual saves through a collective pension fund, rather than individually. The recovery policies of collective funds introduce distortions in labor supply decisions, thereby reducing the risk bearing capacity of participants and causing a decline in ex ante welfare levels. A welfare analysis reveals that the costs of recovery policies (distortions in labor supply decisions) are roughly of the same size as the related benefits (alleviation of the borrowing constraint).

JEL codes: D91, G11, G23

Keywords: Saving, investment, labor supply, life cycle, pension funds.

Email address: r.j.mehlkopf@uvt.nl (Roel Mehlkopf).

1 Preliminary – Please do not quote nor cite. The author thanks Frank de Jong, Lans Bovenberg and the participants of the Netspar lunch seminar series for helpful suggestions.

17 November 2008
1 Introduction

Bodie et al. (1992) have shown that the labor-leisure choice plays an important role in intertemporal saving and investment decision making over the life cycle of an individual. Labor supply flexibility is a blessing in the analysis of Bodie et al. (1992), since income effects in labor supply provide a buffer against income shocks. A negative (positive) income shock causes the marginal utility from working to increase (decrease), thereby increasing (decreasing) labor earnings. Income effects in labor supply thus have a stabilizing effect on the consumption level of the individual over the life cycle, such that the ability to vary labor supply ex post increases the individual’s ex ante appetite for risk taking. The increased risk bearing capacity enables individuals to take greater advantage of the risk premium in financial markets, thereby improving ex ante welfare levels. In this paper I show that labor supply flexibility is much less of a blessing for an individual who saves in a collective pension fund. The recovery policies applied by collective pension funds, by which current gains and losses are levied upon the future earnings of participants through adjustments in contribution and accrual rates, distort the effective wage level against which labor is supplied. The resulting wage differentials introduce substitution effects between consumption and leisure in the labor supply decisions of pension fund participants, thereby distorting labor supply decision making. In this paper we focus on the difference in the labor supply decisions of a pension fund participant and those in the first-best solution and derive the welfare losses resulting from these differences.

This paper shows that the labor supply distortions of recovery policies reduce the risk bearing capacity of individuals. The labor supply distortions of recovery policies induce destabilizing labor supply behavior, such that consumption levels become more volatile and the participant’s appetite for risk taking is reduced. Thus, due to distortions in labor supply choices, the pension fund participant can take less advantage of the risk premium in financial markets (in comparison to the optimal strategy), resulting in a decline in ex ante wel-
fare levels. At the same time our analysis recognizes that recovery policies also have a welfare improving effect for a pension fund participant. By recouping current shocks upon the future earnings of participants, the human capital of a pension fund participant can be exposed to financial market risk. An individual investor, who we assume to be borrowing constraint, is unable to do so because he or she is unable to borrow against his or her human capital. The exposure to risky assets of an individual investor is therefore limited by the size of his or her financial wealth, such that he or she can not optimally take advantage of the risk premium in financial markets. The borrowing constraint for an individual investor is therefore particularly constraining during the beginning of the working period, during which little financial wealth is accumulated. Recovery policies enable pension funds to alleviate the borrowing constraint. The alleviation of the borrowing constraint enables pension fund participants to take more advantage of the risk premium on risky assets during the beginning of the working period, thereby increasing ex ante welfare levels.

Thus, both the individual investor as well as the pension fund participants find are unable to realize the optimal exposure to risky investments, thereby not fully taking advantage of the risk premium in financial markets. Moreover, recovery policies of pension funds feature an important tradeoff with respect to the risk bearing capacity of participants. On the one hand, recovery policies increase the risk bearing capacity of participants due to the alleviation of the borrowing constraint, while on the other hand they reduce the risk bearing capacity due to the labor supply distortions involved. This tradeoff is important for the welfare consequences of pension contracts: the risk bearing capacity determines the extend to which individuals can take advantage of the risk premium in financial markets. On the basis of a welfare analysis this paper reveals that the costs of recovery policies (distortions in labor supply decisions) are roughly of the same size as the related benefits (the alleviation of the borrowing constraint).

Pension funds are characterized by the fact that participants pledge pension contributions to the fund during their working period, for which pension rights
are accrued in return. In order to recover from financial surpluses or deficits, pension funds apply recovery policies, by which current gains or losses are levied upon participants by the adjustment of contribution rates and the value of the corresponding pension rights accrued in return. Negative (positive) financial shocks cause pension fund boards to increase (reduce) contribution rates and to decrease (increase) in the value of corresponding pension rights accrued in return. Current shocks are thus recouped upon participants by taxing or subsidizing the labor earnings of participants over a certain period of time. By taxing or subsidizing labor income, recovery policies distort the effective wage level against which labor is supplied by pension fund participants. On the basis of a model for a stand-alone pension fund, this paper derives that recovery policies result in dramatic deviations in the effective marginal wage level: the unconditional wage differential equals 17% for the default parameters used in this paper.

In the case of large labor market mobility where workers can easily switch between employers, the wage differentials of recovery policies trigger an influx or outflow of workers such that employers find themselves forced to compensate wage differentials through adjustments in salary levels. Thus, if the labor market flexible, not the employees but the employers find themselves paying or receiving the wage differentials of the recovery policies of their pension fund. However, more often than not employees do not have the possibility to evade recovery policies by switching employers. Workers participating in a pension fund that covers a complete sector or country are unable to walk away from their pension contract by switching employers in their sector or country. Furthermore, older workers having pay scales based upon seniority will often have little possibilities to move to another employer. Third, workers with company-specific human capital will find switching employers unattractive.

Adjustments to the value of accrued pension rights are typically made through an indexation policy by which the pension fund board decides, conditional on the funding status, whether the value of pension rights is fully, partially or not at all linked to inflation levels.
tive due to a related wage decline. Throughout the paper we therefore assume that the pension fund participant does not have the possibility to evade the recovery policy of the pension fund by switching employers, such that the wage differentials of recovery policies are fully born by the employees, rather than the employer.$^3$

In this paper we are only interested in the welfare effects resulting from the differences in the labor supply choices of a pension fund participant and the labor supply choices in the first-best strategy. This paper shows that, if designed optimally, the income effects in labor supply are optimal and thus not affected by the pension fund policy. Income effects in labor supply decisions can therefore be ignored in our analysis since they do not distort labor supply choices. However, the labor supply choices of a pension fund participant deviate from the optimal labor supply levels as are result of intra-temporal substitution effects in labor supply. The wage differentials induced by recovery policies affect the reward on labor (i.e. the price of leisure) thereby causing substitution effects between consumption and leisure$^4$ in the labor-leisure choice of the pension fund participant. In our analysis, we therefore solely focus on the intra-temporal substitution effects in labor supply and derive the related welfare effects. In the empirical labor economics literature, intra-temporal substitution effects between consumption and leisure resulting from changes in the wage level are measured by the compensated wage-elasticity of labor supply.$^5$ Recent overviews of Blundell and MaCurdy (1999) and Alesina

$^3$ Obviously, having the wage differentials of recovery policies being born by the employer also leads to undesirable effects. For instance, companies will find it less attractive to engage in risky (but profitable) investment projects if they also have to share in the risk of their corporate pension fund.

$^4$ Leisure is used here in a broad sense of the term, that is any non market (or non traded) activity such as home production, work in the black economy or, indeed, having fun.

$^5$ For an individual who only cares about consumption and leisure, a change in the effective wage level has two effects on labor supply. The first effect is that an increase (decrease) in the effective wage level increases (decreases) the price of
et al. (2005) find a consensus in the empirical labor supply literature that the compensated labor supply elasticity of male workers is low, and that 0.2 is a reasonable model parameter. For women on the other hand, there doesn’t seem to be such a consensus. The median estimate found in the literature for the compensated labor supply elasticity among women is about one. However, due to the large variation in the estimates found in the literatures, a conservative parameter of 0.5 is chosen. Thus, for males (females), a 10% wage differential results in a 2% (5%) difference between the labor supply level of a pension fund participant and the labor supply level in the first-best solution.

In this paper I show that the substitution effects related to the recovery policy have a decreasing effect on the risk bearing capacity of a pension fund participant. Thus, if participating in a collective pension fund, the merits from the ability to vary labor supply ex post are not as large as suggested by the analysis of Bodie et al. (1992). The recovery policy of the pension fund induces destabilizing labor supply behavior, making consumption levels more volatile over the life cycle and thereby causing a decline in the risk bearing capacity of pension fund participants. The allocation to risky assets declines by 22% for a pension fund with half male and half female participants.

Our model takes the perspective of a pension fund participant who participates in the labor force continuously until retirement, which is most appropriate for male workers. Our theoretical framework finds that, for a male worker, the labor supply distortions induced by recovery policies result in a welfare decline of 1.0% in certainty equivalent life time consumption. Our model is

leisure, thereby increasing (decreasing) labor supply through a substitution effect. Second, a wage increase (decrease) raises (lowers) the overall wealth level of the individual, causing the marginal utility of working to decrease which results in lower (higher) labor supply levels as a result of an income effect. The wage differentials of recovery policies only cause substitution effects in labor supply, not income effects. Substitution effects in labor supply are measured by compensated labor supply elasticities: the elasticity of labor supply with respect to changes in the wage level corrected for income effects.
somewhat less appropriate for female workers since it ignores the complexities
of a participation decision. Nevertheless our analysis suggests that the welfare
decline for female workers is much larger. Ignoring participation issues, our
analysis obtains a welfare decline of 3.2% in certainty equivalent life-time
consumption for female workers. The average welfare loss of a participant of
a pension fund with half male and half female participants is then equal to
2.2%.

As previously discussed, recovery policies also have a welfare improving effect
by enabling participants to borrow against their human capital. Bovenberg
et al. (2007) derive that the alleviation of the borrowing constraint results in
a welfare increase of 2.8% of certainty equivalent life time consumption, such
that it can be concluded that the welfare costs (distortions in labor supply) of
recovery policies are roughly of the same size as the related benefits (alleviation
of the borrowing constraint).

This paper contributes to the theory on saving and investing over the life cy-
cle. Most prior work in this field builds on the seminal contributions of Merton
(1969) and Samuelson (1969) and has treated labor earnings as exogenous (Vi-
ceira (2001), Cocco et al. (2005), Gomes and Michaelides (2007)). Bodie et al.
(1992) illustrated that flexible labor supply increases the risk bearing capacity
of individuals and increases ex ante welfare levels. Recent contributions elab-
orate on the work of Bodie et al. (1992). Farhi and Panageas (2007) and Choi
et al. (2008) treat the case where labor supply flexibility is restricted to choice
of an irreversible and indivisible retirement date. Cvitanic et al. (2007) allows
for more general preferences for the investor and the model of Gomes et al.
(2008) features realistically calibrated non-traded labor income. This paper is
the first to show that, for a pension fund participant, labor supply flexibility
also induces effects that reduce the investor’s appetite for risk taking and de-
crease ex ante welfare levels. My result that suboptimal labor supply choices

\[6 \text{ The model and model parameters for the economy and individual preferences in Bovenberg et al. (2007) are the same as in this paper.} \]
result in substantial welfare declines is consistent with Gomes et al. (2008), who find that fixing the labor supply level at a constant level over the working life results in a larger welfare decline than restricting the asset allocation to be constant.

This paper also contributes to the literature discussing the advantages and disadvantages of collective pension funds (Teulings and de Vries (2006), Gollier (2007), Cui et al. (2007), Bovenberg et al. (2007)). The advantages of collective pension funds include lower transaction costs, possibilities to share risks between overlapping and non-overlapping generations, the alleviation of borrowing constraints and overcoming problems related to adverse selection, the incompleteness of financial markets and the bounded rationality of individuals. On the other hand, collective pension funds typically fail to provide tailor-made pension arrangements, apply suboptimal policy rules with respect to consumption smoothing over the life cycle and are often incomplete and not transparent. This paper contributes to our knowledge on the welfare effects of pension fund policies by pointing out that the labor supply distortions caused by recovery policies come at substantial welfare losses. Our findings suggest that the prevention of labor supply distortions is to be taken into account in the design of collective pension contracts. Pension contracts are improved if financial shocks are levied upon participants in a direct way, as is the case in the first-best strategy, rather than through a recovery period. Pension fund policies can be redesigned to reduce labor supply distortions, while at the same time maintaining all other advantages (e.g. alleviation of borrowing constraints, intergenerational risk sharing) that collective defined-benefit pension funds enjoy in comparison to individual (defined contribution) saving schemes.

The structure of the paper is as follows: section 2 introduces the model for the economy and the individual. Section 3 characterizes the first-best decision making rules with respect to labor supply, consumption and investments. Section 4 shows that the individual solution of section 3 can be replicated by a recovery policy of a pension fund. Sections 5 and 6 derive respectively the labor supply distortions and welfare declines resulting from the pension fund
recovery policy. Finally, section 7 concludes.

2 The model

2.1 The economy

The real economy is given by a standard Black-Scholes setting in which the financial market consists of two assets: a riskless asset which pays an instantaneous real net rate of return $r$ and a risky asset of which the real price follows a Brownian Motion with instantaneous volatility $\sigma$ and risk premium $\lambda$, both assumed constant over time. The risk premium represents the expected excess return (over the riskfree rate) per standard deviation of the excess return, also referred to as the Sharpe ratio. Investing in the risky asset introduces uncertainty, but increases the expected return on investments compared to investing in the riskless asset. In the Black-Scholes economy, the present value of a payoff $X(s)$ at time $t$ with $s \geq t$ is equal to $E_t \left[ \frac{M(s)}{M(t)} X(s) \right]$, where $M(t)$ represents the stochastic discount factor at time $t$ and is defined by the following stochastic differential equation:

$$\frac{dM(t)}{M(t)} = -r dt - \lambda dZ(t) \tag{2.1}$$

where $Z(t)$ is a standard Wiener process. Throughout this paper we assume the riskfree rate equal to 2% and the both the volatility as well as the risk premium is assumed to be 20%, such that the expected return of risky investments in excess of the riskfree rate equals 4%.

2.2 The individual

Throughout this paper we consider an individual who works for a period of $R$ years before being retired for a period of $T - R$ years. The labor supply level $h(t)$ of the individual at time $t$ during the working period is a decision
variable, with \( h(t) \geq 0 \) for all \( t < R \). In the retirement period, no labor is supplied, i.e. \( h(t) = 0 \) for \( t \geq R \). The real wage level per unit of labor supply is denoted by \( w \) and is assumed to be constant during the working period. Let the financial wealth of the individual at time \( t \) be denoted by \( F(t) \). The budget constraint on financial wealth is given by the following stochastic differential equation:

\[
dF(t) = \begin{cases} 
(r + x(t)\lambda)F(t)dt + x(t)\sigma F(t)dZ(t) + (h(t)w - C(t))dt & \text{if } t < R \\
(r + x(t)\lambda)F(t)dt + x(t)\sigma F(t)dZ(t) - C(t)dt & \text{if } t \geq R
\end{cases}
\]

where \( x(t) \) represents the fraction of financial assets invested in the risky asset at time \( t \) and where \( C(t) \) represents consumption level of the individual at time \( t \). The first and second term on the right-hand-side of equation (2.2) represent respectively the drift and the diffusion component of the return on financial wealth, whereas the third term represents the net savings of the individual at time \( t \), where net savings represent earnings minus consumption. The initial financial wealth of the individual is equal to zero \( (F(0) = 0) \) and the model abstracts from a bequest motive, such that the individual finds it optimal to consume all wealth during the life cycle \( (F(T) = 0) \). With respect to the budget constraint, the individual maximizes time-separable expected utility from consumption and labor supply:

\[
U(t) = \mathbb{E}_t \left[ \int_t^T e^{-\beta(s-t)}u(C(s), h(s))ds \right]
\]

for \( 0 \leq t \leq T \), where \( \beta \) represents the individual’s rate of time preference and where the instantaneous utility function at time \( t \) is given by:

\[
u(C(t), h(t)) = \frac{1}{1-\gamma} \left( C(t) - \frac{\epsilon}{\epsilon + 1} h(t)^{\eta+1} + \eta \right)^{1-\gamma}
\]

where \( \eta \) represents a constant that will be defined at a later point in this section, and where \( \epsilon \) represents the intratemporal labor supply elasticity with respect to the wage level. Thus, an increase (decrease) in the wage level by one percent results in a rise (drop) in the labor supply level of \( \epsilon \) percent. Originating from Greenwood et al. (1988), the specification in (2.4) features the
property that income effects in labor supply decision making are eliminated from labor supply decision. This is rather convenient for our analysis, since the recovery policies of pension funds do not introduce additional income effects in labor supply, as will be shown formally in section 4. Our analysis therefore focuses on intra-temporal substitution effects in labor supply, which are fully responsible for the difference between the labor supply choices of an individual investor and a those of a pension fund participant. Given that that income effects are omitted from our analysis, the parameter $\xi$ has the interpretation of the compensated wage-elasticity of labor supply, i.e. the wage-elasticity of labor supply corrected for income effects.

Thus, it is important to bear in mind that the individual preferences in equations (2.3) and (2.4) do not describe the actual preferences of the individual. Instead, our specification only capture the effects (substitution effects in labor supply) responsible for the differences in the labor supply decisions between a pension fund participant and an individual investor. This allows us the derive the labor supply distortions and the related welfare effects of recovery policies in a simple and transparent way.

Due to the absence of income effects in labor supply in our model, labor supply decisions are fully driven by the intra-temporal tradeoff between consumption and leisure and are thereby determined by the effective marginal wage level (i.e. the price of leisure) against which labor is supplied. In absence of labor supply distortions, the effective marginal wage level during the working period is given by $w$ such that labor supply is given by $^7$

$$h(t) = h^* = w^\xi$$  \hspace{1cm} (2.5)

for $t < R$, and it follows that labor supply levels are constant over the life-cycle. Negative labor supply levels are ruled out by the model as long as effective marginal wage levels are nonnegative.

Section 6 compares welfare losses resulting from labor supply distortions in

\footnote{See Appendix A for a formal proof.}
pension funds for various choices for the elasticity of labor supply parameter $\epsilon$. In order to prevent that a change in the labor supply elasticity parameter $\epsilon$ affects the interpretation of parameter $\gamma$, parameter $\eta$ in the utility function in equation (2.4) is defined as:

$$\eta = \frac{\epsilon}{\epsilon + 1} \left( h^* \right)^{\frac{\epsilon + 1}{\epsilon}} = \frac{\epsilon}{\epsilon + 1} w^{\epsilon + 1} \tag{2.6}$$

where $h^*$ is defined by equation (2.5). From substitution of equation (2.6) into equation (2.4) it follows that in absence of labor supply distortions, $\gamma$ represents the level of relative risk aversion with respect to marginal consumption, regardless of the choice for the parameter of labor supply elasticity $\epsilon$. Thus, if the labor supply is optimal, the definition of $\eta$ in equation (2.6) ensures that the level of relative risk aversion with respect to consumption is equal to $\gamma$, regardless of the choice of labor supply elasticity $\epsilon$. If the labor supply choice is suboptimal, then the level of relative risk aversion over consumption will be close to $\gamma$, but not equal. After all, the individual has relative risk aversion $\gamma$ over a composite good consisting of consumption and some function of leisure, where the function of leisure is equal to zero if labor supply is optimal.

Throughout this paper we assume a rate of time preference $\beta$ of 2% and a parameter of relative risk aversion $\gamma$ of 5. The compensated wage-elasticity of labor supply $\epsilon$ is assumed 0.2 for males and 0.5 for females.

3 The optimal solution

Bodie et al. (1992) provide a general solution for the simultaneous optimization of labor supply, consumption and investment choices over the life cycle. Our model is a special case of the more general treatment of Bodie et al. (1992), since the absence of wealth effects causes intra-temporal labor supply decision making to be independent of the consumption-investment strategy of the individual. With the labor supply choice solved in equation (2.5), the remaining decision making problem reduces to a consumption-investment decision mak-
ing problem, as treated by Merton (1969) and Samuelson (1969). For completeness, this section provides a short overview of the optimal consumption-investment solution strategy.

3.1 The special case without risk taking ($x = 0$)

In absence of risk taking, the optimal consumption path is smooth over the life cycle and is fully characterized by its growth rate. The growth rate of the optimal consumption profile is given by:

$$\frac{d \log(C(t))}{dt} = \frac{r - \beta}{\gamma}$$

indicating that the consumption path is increasing (decreasing) over the life-cycle if the interest rate $r$ is larger (smaller) than the individual’s rate of time preference $\beta$. The extent to which this difference affects the slope of the consumption profile is determined by the willingness of the individual to substitute consumption over time intertemporally, i.e. the elasticity of intertemporal substitution of consumption, $1/\gamma$. \(^8\)

Figure 8 illustrates the case where earnings during the working period are normalized to unity. Recall that earnings in our model are flat over the working period, since both wages and labor supply (specified in equation (2.5)) are constant. The interest rate $r$ equals the individual’s rate of time preference $\beta$ for our model parameters such that the consumption profile is flat over the life-cycle. During the working period, the labor income of the individual exceeds consumption level, such that the individual has positive savings. These savings result in the accumulation of financial wealth, required to finance consumption in the retirement period in which no earnings are obtained and financial wealth is decumulated.

\(^8\) The individual has time-separable preferences with a constant level of relative risk aversion $\gamma$ over consumption, such that the elasticity of intertemporal substitution of consumption is constant over the life-cycle and is equal to $1/\gamma$. 

13
Fig. 1. Labor income $h(t)w(t)$, consumption $C(t)$, financial wealth $F(t)$ and human wealth $H(t)$ in absence of risk taking for the model parameters. Earnings during the working period are normalized to unity.

### 3.2 The general solution

In the presence of risk taking, the optimal investment rule is characterized by the property that the individual invests a fixed fraction $\lambda$ of the total wealth $W(t)$ in risky assets during the complete life cycle, where the total wealth of the individual at time $t$ equals the sum of human wealth $H(t)$ and financial wealth $F(t)$ and

$$W(t) = H(t) + F(t) \quad (3.2)$$

The human wealth $H(t)$ at time $t$ represents the market value of earnings in the remaining working period. Given that, in absence of labor supply distortions, labor supply is constant over the working period, the future earnings of the
individual are deterministic and human capital is given by:

\[
H(t) = \begin{cases} 
E_t \left[ \int_{t}^{\infty} M(s) \frac{dW}{W} ds \right] & \text{for } 0 \leq t < R \\
0 & \text{for } R \leq t < T
\end{cases}
\]  

(3.3)

where \( h(t) = h^* \) in the optimal solution strategy.

The individual consumption profile becomes stochastic in the presence of risk taking and is characterized by the way in which financial shocks are smoothed over consumption levels. In the optimal consumption rule, a financial shock at time \( t \) is levied proportionally on all consumption levels in the remaining lifetime, such that shocks to remaining consumption levels are proportional to wealth shocks. Given that the elasticity of intertemporal substitution of consumption is and have the same elasticity with respect to the current shock:

\[
\frac{d\log C(s)}{d\log W(t)} = 1
\]  

(3.4)

for all \( t \) and for all \( s > t \).

Figures 2(a) and 2(b) illustrate respectively the consumption and financial wealth processes for the general case with risk taking for an example scenario path (solid lines) and the 90\% confidence intervals. In the example scenario, the returns on risky investments in are below expectations the first half of the working period, such that consumption and wealth levels in the presence or risk taking drop below their respective levels in absence of risk taking (compare to Figure 8). In the second half of the working period and in the retirement period, the returns on risky investments are above expectations, such that consumption and wealth levels in the example scenario path recover and even exceed their respective levels in absence of risk taking. Since investments in the risky asset increase the expected return on investments, the growth rate of the expected consumption path (i.e. the 50\% quantile of the distribution of the consumption level) is larger than the growth rate of consumption in absence of risk taking.

Figures 2(c) and 2(d) illustrate respectively the total wealth and the amount
Fig. 2. An example scenario path (solid line) and 90% confidence intervals for Consumption $C(t)$, financial wealth $F(t)$ and the amount invested in risky assets $(x(t)F(t))$ in the general case with risk taking. Earnings during the working period are normalized to unity.

invested in risky assets over the life cycle. The figures illustrate that the amount invested in risky assets is a fixed fraction of total wealth, 20% for the parameters applied in this paper. It is important to note that, in the beginning of the working period, the amount invested in risky assets exceeds the financial wealth of the individual. The optimal solution can therefore only be executed if the individual is able to borrow against his or her human capital. An individual saving through an individual retirement savings account is unable to do so, but the next section explains how the borrowing constraint is alleviated if the individual saves in the collective pension fund.
4 The pension fund policy

This section introduces a stylized model for a pension fund applying a recovery policy to recoup financial shocks on its participants. We take the perspective of a single participant in the fund and abstract from intergenerational transfers. The characteristics of the economy and the individual participant are those introduced in section 2. Assuming exogenous labor supply, we are able to perfectly replicate the individual solution derived in section 3. With labor supply assumed exogenous, the pension fund participant does not respond to the labor supply incentives provided by the pension fund policy but instead supply a constant amount of labor over the life-cycle equal to the labor supply level $h^*$ in equation (2.5). The assumption of exogenous labor supply of the participant will be relaxed in section 5, where the labor supply distortions of the recovery policy are examined.

The pension fund is characterized by the fact that pension rights are accrued in return for contribution payments to the fund during the working period. Pension rights accumulated during the working period are transferred into benefit payments in the retirement period. The pension fund has three policy instruments to its disposal:

- The contribution policy, which adjusts the current contribution level.
- The indexation policy, which adjusts the market value of pension rights accrued at present and in those accumulated in the past, where the proportional adjustment to previous accumulated pension rights and pension rights accrued at present is the same.\(^9\)

\(^9\) Pension funds typically aim to offer benefit levels that are linked to price inflation or wage inflation. Protection against inflation is often conditional on the funding status, such that pension accruals are not or only partially indexed if returns on risky investments are below expectations. Thus, adjustments in the accrual rates are governed by the ‘indexation’ level provided by the pension fund. If indexation is provided unconditionally, for instance in a defined benefit scheme, only the contribution policy and the investment policy can be used to manage the financial status.
• The investment policy, which adjusts the allocation between risky and riskless assets.

The pension fund policy is optimal if it exposes the participant with the optimal risk exposure and if financial shocks are levied upon future consumption levels optimally. The contribution policy optimally adjusts the consumption levels of the participant during the working period, whereas the indexation policy optimally adjusts consumption levels in the retirement period. The investment policy is designed such that the participant has the optimal exposure to risky investments.

The appendix shows that it is possible to replicate the optimal strategy with these three policy instruments. In the optimal pension policy, there is an ex ante fair relation between the contribution rate \( \pi(t) \) (that is: the fraction of earnings pledged to the pension fund) per unit of labor supply and the corresponding market value of the rate \( \alpha(t) \) of pension accruals accrued in return

\[
E_0 \left[ \frac{M(t)}{M(0)} \pi(t) \right] = E_0 \left[ \frac{M(t)}{M(0)} \alpha(t) \right]
\]

for all \( 0 < t < R \). While being fairly related ex ante, contributions and the related market value of pension accruals deviate from each other ex post. If the market value of pension accruals exceeds the contribution level, the pension fund provides a net subsidy on the labor supply of the participant, whereas a net tax is levied on labor supply in the opposite case. The market value of the net tax or net subsidy levied upon the future labor income of the participant is referred to as the surplus \( S(t) \) of the participant:

\[
S(t) = \begin{cases} 
\int_t^R E_t \left[ \frac{M(s)}{M(t)} \omega h^*(\alpha(s) - \pi(s)) \right] ds & \text{if } t < R \\
0 & \text{if } t > R
\end{cases}
\]

where \( \alpha(s) - \pi(s) \) represents the wage differential per unit of labor supply at time \( s \). The surplus equals zero during the retirement period and it follows from (4.1) that the initial surplus of the participant equals zero as well. The surplus of the fund.

18
wealth of a pension fund participant is equal to the sum of human wealth \( H(t) \) (as defined in equation (3.3)), the market value of accumulated pension rights, denoted by \( A(t) \), and the surplus \( S(t) \)

\[
W(t) = H(t) + A(t) + S(t)
\]  
(4.3)

The optimal contribution policy proportionally adjusts the consumption levels of the participant during the working period according to the optimal consumption smoothing rule in equation (3.4). That is: the contribution policy is optimal if all remaining disposable income levels of a working participant (i.e. the fractions \( 1 - \pi(s) \) \((s > t)\) of income that are not pledged to the pension fund) are adjusted proportionally to a wealth shock at time \( t \):

\[
\frac{d\log(1 - \pi(s))}{d\log W(t)} = 1
\]  
(4.4)

for all \( t < R \) and for all \( t < s < R \). Thus, in absence of future shocks, all contributions levels in the remaining working period of the pension fund are adjusted by the same percentage after a shock in wealth.

The indexation policy should optimally adjust benefit levels in the retirement period. An indexation adjustment affects the market value of pension rights accrued at present and in those accumulated in the past proportionally equally. Let \( \xi(t) \) denote the cumulative indexation level of the pension fund at time \( t \), i.e. the cumulative indexation provided by the pension fund over the period \([0, t]\). The indexation policy of the pension fund causes an adjustment in the indexation level to result in a proportional change in both the market value of accumulated pension rights as well as in the market value of future accrual rates, i.e:

\[
\frac{d\log A(t)}{d\log \xi(t)} = 1
\]  
(4.5)

for all \( t \), and

\[
\frac{d\log a(s)}{d\log \xi(t)} = 1
\]  
(4.6)

for all \( t \) and all \( s > t \). Indeed, the indexation policies of pension funds have the effects described in equations (4.5) and (4.6). By a current adjustment of the indexation level, the value of pension rights accumulated in the past
as well as the value of those to be accumulated in the future are all affected by the same percentage. The Appendix shows that the optimal consumption smoothing rule in equation (3.4) is replicated if cumulative indexation levels are adjusted proportionally to wealth shocks

\[
\frac{d\log \xi(s)}{d\log W(t)} = 1
\]  

(4.7)

for all \( t \) and all \( s > t \). The indexation rule in equation (4.7) ensures that a change in wealth results in a proportional adjustment of the market value of pension rights accrued in the past and in the future. If benefit levels are proportional to value of pension rights accumulated over the working period as a whole, then it is accomplished that changes in the benefit level of a pension fund participant are proportional to wealth shocks.

The optimal investment strategy of section 3 is replicated by investing a fixed fraction \( \frac{\lambda}{\sigma} \) of total wealth \( W(t) \) of the participant in risky assets. With the risk exposure and the consumption smoothing rules optimized, the wealth level \( W(t) \) of a pension fund participant coincides with the total wealth level in the optimal strategy at all times, such that no additional wealth effects are introduced by the pension fund policy. This property of the pension fund policy justifies the absence of income effects in labor supply in our model, since the income effects in labor supply of the pension fund participant coincide with those in the optimal strategy.

The pension fund policy affects the participant in a direct way and in an indirect way. Directly, the value of pension rights accumulated in the past is affected by a current shock. Indirectly, a current shock affects future contribution rates as well as the market value of future accrual rates, i.e. the surplus. This indirect way in which the pension fund recoups a financial shocks on its participants is referred to as the recovery policy. A relatively old working participant, with a relatively short remaining working period and a relatively large amount of accumulated pension rights, will be mostly affected through the direct channel. Young workers, with a long remaining working period and with a relatively low amount of pension rights accumulated, are mainly hurt
through the recovery policy of the fund.

Figures 3(a) and 3(b) illustrate the contribution rate $\pi(t)$ and the market value of pension accruals $\alpha(t)$ over the working life cycle of the pension fund participant for the example scenario path (solid line) and the respective 90% confidence intervals. In the first half of the working period, the surplus of the pension fund participant is negative such that contribution levels exceed the corresponding values of pension rights accrued in return. This is due to both an increase in the contribution levels as well as a decrease in the accrual rates. In the first half of the working period the pension fund effectively levies a net tax on labor earnings. In the second half of the working period, the surplus of the pension fund participant is positive such that the value of pension rights accrued exceeds the corresponding price paid in the form of pension contributions. The fund thus provides a net subsidy on labor earnings in the second half of the working period.

The solid line in Figure 3(c) shows the value of accumulated pension rights $A(t)$ over the life cycle of the individual participant for the same example scenario path as introduced in Figure 2. The figure illustrates that pension rights process is roughly increasing in the working period and decreasing in the retirement period. The solid line in Figure 3(d) shows the surplus $S(t)$ over the life cycle for the example scenario path. For the example simulation path, the surplus of the participant is negative during the first half of the working period since the losses on risky investments need to be recouped in the remaining life time of the participant. During the second half of the working period, the financial status of the fund recovers.

5 Labor supply distortions induced by the recovery policy

Labor supply distortions result from the wage differential caused by the recovery policy. The effective marginal wage level of the pension fund participant $\tilde{w}$ per unit of labor supply at time $t$ equals the wage level $w$ minus pension
Fig. 3. The value of pension rights accumulated $A(t)$, the surplus $S(t)$, the contribution rate $\pi(t)$ and the accrual rate $\alpha(t)$ of the pension fund participant for the example scenario path (solid line) and the corresponding 90% confidence interval (dotted lines).

contributions plus the market value of accrued pension rights:

$$\hat{w}(t) = w + w(t)\pi(t) - w\alpha(t) = w(t) (1 - \pi(t) + \alpha(t))$$

for $t < R$.

Figure 4 illustrates the example scenario path (solid line) and 90% confidence interval (dotted lines) for the effective marginal wage level of a pension fund participant. The figure shows that, in the example scenario path, the pension fund policy decreases the effective marginal wage level of the pension fund participant in the first half of the working period. This is caused by both increasing contribution levels (see Figure 3(a)) and decreasing corresponding values for the pension rights accrued in return (see Figure 3(b)). Thereby, the pension fund in effect levies a net tax on labor income in the first half of the working period. During the second half of the working period, the
Fig. 4. The effective wage level $\tilde{w}(t)$ of a pension fund participant at time $t$ for the example scenario path (solid line) and the 90% confidence interval (dotted lines). The example scenario path corresponds to the scenario path in previous figures. The wage level in absence of labor supply distortions is normalized to unity. The average distortion to the wage level over the working life as a whole is equal to 16.9%.

The financial situation of the pension fund recovers and the effective wage level of the pension fund participant exceeds the corresponding wage levels of an individual in an actuarially fair pension plan, such that the pension fund in effect provides a net subsidy on labor income in the second half of the working period. The 90% confidence interval in Figure 4 illustrate that recovery policies of pension funds result in substantial distortions in wage level: the average percentage by which the wage level is affected over the working life as a whole equals 16.9%.

Assuming pension fund participants are able to continuously save and borrow outside the pension fund, participants can sell their accrued pension rights instantaneously in the financial market. The extend to which the labor supply decisions of a pension fund participant respond to the wage differentials is determined by wage-elasticity of labor supply as measured by parameter $\epsilon$. From the labor supply decision rule in equation (2.5) it follows that the labor supply level $\hat{h}(t)$ of a pension fund participant with access to financial market

---

10 This is possible because we assume a complete financial market. Pension fund participants may find themselves unable to sell accrued pension rights if the financial market is incomplete.
Fig. 5. The labor supply choice of a pension fund participant $\hat{h}(t)$ of a pension fund participant at time $t$ for the example scenario path (solid line) and the 90% confidence interval (dotted lines) for various levels of the parameter of labor supply elasticity $\epsilon$. The labor supply choice of the corresponding individual investor is normalized to unity. The average labor supply distortion is 3.0% in the case where $\epsilon=0.2$ and it increases to 7.9% in the case where $\epsilon=0.5$.

is equal to

$$\hat{h}(t) = \hat{w}(t)^\epsilon = h^*(1 - \pi(t) + \alpha(t))^\epsilon$$

(5.2)

for $t < R$, where $h^*$ denotes the optimal individual labor supply rule as given by equation (2.5). The labor supply level of the pension participant is positive at all times since recovery policy never causes the effective marginal wage level to become negative$^{11}$. The optimal solution features the property that the exposure of labor supply levels to economic shocks is uniform across age, i.e. an economic shock affects the labor supply level of respectively a young and an old participant by the same percentage. This result is driven by the fact that the exposure of the effective wage level $\hat{w}(t)$ of the pension fund participant is uniform across age.

Figure 5(a) illustrates the labor supply distortions for male workers (for whom a compensated wage-elasticity of labor supply of $\epsilon = 0.2$ is assumed) whereas Figure 5(b) illustrates the case for females (for whom $\epsilon = 0.5$ is assumed). For male workers the average labor supply distortion is equal to 3.0%, whereas it

$^{11}$This follows from the fact that the disposable income rates $1 - \pi(t)$ as well as the market value of pension accruals $\alpha(t)$ follow a Geometric Brownian Motion and thus remain positive at all times.
is substantially higher for female workers: 7.9%.

For the derivation of equation (5.2) the pension fund participant was assumed to have full access to financial markets. The alternative assumption, where the participant is unable to save or borrow outside the pension fund, yields exactly the same result. In absence of access to financial markets, the value of accrued pension rights for the participant is given by the monetary equivalent of the utility gained from the pension rights received in return for supplying labor. That is: the constrained participant is indifferent between receiving pension accruals and the monetary equivalent of these pension accruals. It is easy to see that the monetary equivalent of utility gained from pension accruals equals the market price of those pension accruals. After all, the pension fund policy perfectly replicates the first-best solution in which the marginal rates of substitution (the ratios of marginal utilities across time) are perfectly reflected by market prices, such that the monetary equivalent of utility gained from pension accruals is equal the corresponding market price.

6 Welfare losses resulting from labor supply distortions

To derive the welfare losses from labor supply distortions, we simply compare the welfare levels under optimal labor supply (given by \( h^* \)) and suboptimal labor supply (given by \( \hat{h}(t) \)). In the case of suboptimal labor supply, the savings and investment rules are re-optimized\(^{12}\).

Figure (6) that the optimal fraction of wealth invested in risky assets is decreasing in labor supply distortions. This result is intuitively explained by

\(^{12}\) For simplicity it is imposed that the fraction of total wealth invested in risky assets is constant over time in the re-optimization. Without this simplification, the re-optimized asset allocations will voluntarily impose a short selling constraint if the elasticity of labor supply is large. After all, avoiding short positions in the asset allocation prevents labor supply distortions. However, given that pension funds usually apply uniform policies, such a re-optimization wouldn’t be realistic.
observing that substitution effects (introduced by the recovery policy) have the opposite effect on the labor supply choices of individual as income effects, i.e. income effects and substitution effects in labor supply work against each other. Thereby, the substitution effects induced by the recovery policy offset stabilizing income effects in labor supply, thereby increasing the volatility of consumption levels and reducing the individual’s appetite for risk taking. Thus, labor supply flexibility has two opposite effects on the risk bearing capacity of a pension fund participant. As shown by Bodie et al. (1992), income effects in labor supply causes the flexibility in labor supply to function as a buffer against income shocks, thereby increasing the risk bearing capacity of the participant. On the other hand, substitution effects in labor supply, introduced by the recovery policy of the pension fund, reduce the appetite for risk taking. For male workers (with $\epsilon=0.2$) the decline in investments in risky assets as a result of substitution effects is relatively modest at 6%, whereas the 37% decline for female workers (with $\epsilon = 0.5$) is large. The allocation to risky assets declines by 22% for a pension fund with a population consisting for 50% of males and 50% of female workers, compared to the allocation in the optimal solution.

The solid line in Figure 7 shows, for various levels of the elasticity of labor supply $\epsilon$, the welfare decline in terms of the certainty equivalent of lifetime
Fig. 7. The % change in the certainty equivalent consumption level (ceteris paribus the leisure level, which is fixed at the levels of the first-best solution given by (2.5) for the calculation of the certainty equivalents) over the life cycle for various levels of the elasticity of labor supply $\epsilon$.

The change in the certainty equivalent consumption level (ceteris paribus the leisure level, which is fixed at the levels of the first-best solution given by (2.5) for the calculation of the certainty equivalents) over the life cycle for various levels of the elasticity of labor supply $\epsilon$.

physical consumption$^{13}$ over the life cycle resulting from labor supply distortions. For male workers ($\epsilon=0.2$), the welfare loss is relatively modest at 1.0%. However, our model suggests that the welfare losses for female workers ($\epsilon = 0.5$) are large: we find decline of 3.2%. For a pension fund with 50% male and 50% female workers, this implies that the welfare loss for the average participant equals 2.1%.

Recovery policies not only have welfare costs but also have an important welfare improving aspect. Recovery policies enable individuals to borrow against their human capital. As illustrated in Figure 2, the amount invested in risky assets in the optimal individual investment strategy exceeds the amount of financial wealth, thereby requiring the individual to be able to borrow against his or her human capital. Individual investors are typically unable to borrow against their human capital and are thus not optimally exposed to financial market risks. Bovenberg et al. (2007) show that borrowing constraints result in a welfare decline of 2.8% of certainty equivalent consumption over the life cycle for the parameters used in this model. The recovery policies of pension funds

$^{13}$Leisure level are held fixed (at the levels of the first-best solution as given by equation (2.5)) for the calculation of certainty equivalents
alleviate the borrowing constraint for individuals, thereby improving welfare levels. The welfare analysis in this paper reveals that the costs of the recovery policy of the pension fund (labor supply distortions) are roughly of the same size as the corresponding benefits (alleviating the borrowing constraint).

7 Conclusions

On the basis of a theoretical framework, this paper provides a welfare analysis of the recovery policies of pension funds and shows that the benefits (enabling individuals to borrow against their human capital) are roughly of the same size as the related costs (distortions in labor supply). Our results therefore suggest that the prevention of labor supply distortions is to be taken into account in the design of collective pension contracts. Pension contracts are improved if financial shocks are levied in a direct way upon participants, as is the case in the first-best strategy, rather than through a recovery period. This paper suggests that pension fund policies can be redesigned to reduce labor supply distortions, while at the same time maintaining all other advantages (e.g. alleviation of borrowing constraints, intergenerational risk sharing between non overlapping generations) that collective defined-benefit pension funds enjoy in comparison to individual (defined contribution) saving schemes.

References


Z. Bodie, R.C. Merton, and W.F. Samuelson. Labour supply flexibility and


P.A. Samuelson. Lifetime portfolio selection by dynamic stochastic program-


## A Derivations

**Derivation of equation (2.5)**

The intra-temporal labor supply decision solves from maximizing intratemporal utility

\[
\frac{1}{1-\gamma} \left( C(t) - \frac{\epsilon}{\epsilon + 1} h(t) \frac{w(t+1)}{w(t)} + \eta \right)^{1-\gamma} \tag{A.1}
\]

i.e. the individual maximizes

\[
C(t) - \frac{\epsilon}{\epsilon + 1} h(t) \frac{w(t+1)}{w(t)} + \eta \tag{A.2}
\]

Normalizing total time spent on labor and leisure to unity, and denoting the wealth spent on both consumption and leisure at time \( t \) by \( y(t) \), the intratemporal budget constraint is given by:

\[
y(t) = C(t) + (1 - h(t))w \tag{A.3}
\]

where \( C(t) = h(t)w \). Substitution of the budget constraint in the objective function yields that the individual maximizes

\[
y(t) - (1 - h(t))w - \frac{\epsilon}{\epsilon + 1} h(t) \frac{w(t+1)}{w(t)} + \eta \tag{A.4}
\]

The first-order condition with respect to the labor supply yields the optimal labor supply decision

\[
h(t) = w^\epsilon \tag{A.5}
\]
which is independent of total wealth \( y(t) \) spent at time \( t \). Labor supply elasticity with respect to the real wage level is given by:

\[
\frac{d \log h(t)}{d \log w} = \frac{d \log w^e}{d \log w} = \epsilon
\]

such that the intra-temporal labor supply elasticity with respect to the wage level is equal to \( \epsilon \).

Derivation of equations (4.4) and (4.7)

Let the premium level per unit of labor supply be denoted by \( w \pi(t) \), where the premium ratio \( \pi(t) \) is stochastic and represents the fraction of income pledged to the pension fund at time \( t \). In return for their contributions, participants accrue pension rights at a rate of \( w \frac{\xi(t)}{\xi(0)} a(t) \) per unit of labor supply at time \( t \), where \( a(t) \) is a non-stochastic base accrual rate at time \( t \) and where \( \xi(t) \) represents the cumulative indexation level at time \( t \), i.e. the cumulative indexation on pension accruals provided by the pension fund over the period \([0, t]\).

An adjustment \( d \xi(t) \) to the indexation level at time \( t \) proportionally adjusts the value of pension rights accrued at present as well as those accumulated in the past, where the relative adjustment to past accruals is equal to the relative adjustment to present accruals. Let adjustments to the indexation level be fully driven by the return on risky investments, such that the indexation process is a martingale under the risk neutral measure, i.e.

\[
E_t \left[ \frac{M(s)}{M(t)} \xi(s) \right] = \xi(t)
\]

for all \( s > t \). Equation (A.7) reflects the fact that the pension fund cannot create market value (i.e. increase the market value of accrued pension rights) by investing in risky assets\(^{14}\). As a result, the market value of pension rights \( w \alpha(t) \) accrued per unit of labor supply at time \( t \) equals

\[
w \alpha(t) = w \frac{\xi(t)}{\xi(0)} a(t)
\]

\(^{14}\) See Ponds (2003) for a more elaborate evaluation.
since the value of the amount of pension rights $\xi(t) a(t)$ accrued at time $t$ is unaffected by indexation adjustments (that is: investments in risky assets) after time $t$. The dynamics of the market value of accumulated pension rights $A(t)$ at time $t$ are given by

$$ dA(t) = \begin{cases} A(t) \frac{d\xi(t)}{\xi(t)} + wh(t)\alpha(t) dt & \text{if } t < R \\ A(t) \frac{d\xi(t)}{\xi(t)} - b(t) dt & \text{if } t > R \end{cases} $$

(A.9)

where $A(0) = 0$ and $A(T) = 0$. The first term on the right-hand-size of equation (A.9) reflects the indexation of pension rights accumulated in the past, whereas the second term reflects the accumulation (decumulation) of pension rights during the working period (retirement period).

If the optimal individual consumption levels are perfectly replicated by the pension policy, there is no incentive for pension fund participants to save or borrow outside the pension fund and consumption levels are given by

$$ C(t) = \begin{cases} h^* w (1 - \pi(t)) & \text{if } 0 \leq t < R \\ b(t) & \text{if } 0 \leq t < R \end{cases} $$

(A.10)

Substitution of equation (3.4) into equation (A.10) yields

$$ \frac{d\log(wh^*(1 - \pi(s)))}{d\log W(t)} = 1 \text{ for } s > t, s < R $$

(A.11)

$$ \frac{d\log b(s)}{d\log W(t)} = 1 \text{ for all } t $$

(A.12)

The first line implies that disposable income levels (income after pension contributions) should be proportionally adjusted to wealth shocks

$$ \frac{d\log (1 - \pi(s))}{d\log W(t)} = 1 $$

(A.13)

for $s > t, s < R$, as stated by equation (4.4). The second line implies that benefit levels should be adjusted proportionally to wealth shocks

$$ \frac{d\log b(s)}{d\log W(t)} = 1 $$

(A.14)
for \( s > t, s > R \). In the first-best consumption rule it holds that benefit levels are proportional to wealth levels, such that the benefit payments to pension fund participants should be proportional to the market value of accumulated pension rights:

\[
\frac{\text{dlog} b(s)}{\text{dlog} A(s)} = 1 \quad \text{(A.15)}
\]

for \( s > R \). Combining equations (A.14) and (A.15), the optimal pension policy should ensure that

\[
\frac{\text{dlog} A(s)}{\text{dlog} W(t)} = 1 \quad \text{(A.16)}
\]

for \( s > t, s > R \), and it follows from the second line in equation (A.9) that:

\[
\frac{\text{dlog} A(s)}{\text{dlog} \xi(s)} = 1 \quad \text{(A.17)}
\]

such that equation (A.16) is realized by setting indexation levels proportional to wealth shocks:

\[
\frac{\text{dlog} \xi(s)}{\text{dlog} W(t)} = 1 \quad \text{(A.18)}
\]

for all \( s > t \), from which equation (4.7) follows directly.